EXPERT REPORT OF DR. ARI J. STERN

***Wise v. Missouri*, 2516-CV29597 (Circuit Court of Jackson County, Missouri)**

Amended December 30, 2025

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I. Introduction and Qualifications

1. My name is Ari Stern, Ph.D., and I am a Professor of Mathematics at Washington University in St. Louis ("WashU"). I joined the faculty of WashU in 2012, was awarded tenure in 2018, and was promoted to the rank of (full) Professor in 2024.

2. I hold a Ph.D. in Applied and Computational Mathematics from the California Institute of Technology (better known as "Caltech"), as well as an M.A. in Mathematics of Finance and a B.A. in Mathematics, both from Columbia University. Before arriving at WashU, I was a postdoctoral researcher and lecturer at the University of California, San Diego. My professional CV is attached as Appendix 5.

3. My academic research focuses on computational mathematics and geometry, specifically using geometry to develop and analyze algorithms for scientific computing and other applications. I have authored or coauthored over 30 peer-reviewed publications, which have been published in leading journals in computational mathematics (*Foundations of Computational Mathematics*, *Mathematics of Computation*, *SIAM Journal on Numerical Analysis*, *SIAM Journal on Scientific Computing*, *Numerische Mathematik*) as well as journals focusing on applications in physical, biological, and social sciences (*Multiscale Modeling and Simulation*, *European Journal of Applied Mathematics*, *Journal of Nonlinear Science*, *Computers and Chemical Engineering*, *Science Translational Medicine*, *Games and Economic Behavior*). I am an Associate Editor on the editorial boards of three journals: *Foundations of Computational Mathematics*, *International Journal of Numerical Analysis and Modeling*, and *Geometric Mechanics*. My research activities have received continuous grant support since 2011, including grants from the National Science Foundation, National Institutes of Health, and Simons Foundation.

4. Since 2017, I have been involved in applications of computational mathematics to electoral redistricting analysis, for which the algorithms have similar "geometric" ingredients to those I study in my primary line of research (e.g., both involve the splitting of large shapes into smaller ones). This work has resulted in one peer-reviewed article on computational algorithms for detecting communities of interest[1], several technical reports, testimony to state redistricting commissions, and an *amicus* brief to the Supreme Court of the United States[2].

5. I have been an active user of the open-source *GerryChain* redistricting-analysis software since it was first developed in 2018 by a team including my then-Ph.D.-student Dr. Mary Barker. I have regularly interacted with the MGGG Redistricting Lab (led by Prof. Moon Duchin at the University of Chicago), which continues to develop and maintain *GerryChain* and associated software tools. The MGGG Redistricting Lab lists me as a "key collaborator," and I have contributed algorithms, code, and data to various projects.

6. I previously served as an expert in *New York Communities for Change et al. v. County of Nassau et al.*, Case No. 602316/2024 (Sup. Ct. Nassau Cty., 2025). For that case, I performed computational redistricting analysis, submitted three expert reports, was deposed, and testified as an expert witness in court. The parties ultimately settled that case during trial.

II. Scope of Work

7. I have been asked by counsel representing the *Wise* plaintiffs in this case to analyze the Missouri FIRST Map (enacted as H.B. 1 in the September 2025 special session of the Missouri General Assembly), focusing on the region covered by the newly enacted

[1] Chambers, E., Duchin, M., Edmonds, R. A. C., Edwards, P., Matthews, J., Pizzimenti, A. E., Richardson, C., Rule, P., & Stern, A. (2022). Aggregating Community Maps. In *SIGSPATIAL '22: Proceedings of the 30th International Conference on Advances in Geographic Information Systems*, Paper No. 27, 12 pages, ACM Press, New York.
[2] Brief for Mathematicians et al. as *Amici Curiae* at 19–20, *Rucho v. Common Cause*, 139 S. Ct. 2484 (2019).

Congressional District 4 (“CD4”) and Congressional District 5 (“CD5”) and the boundary dividing these two districts.

8. Counsel informed me that the Missouri Constitution requires that Congressional districts be “as compact . . . as may be.” Mo. Const. art. III, § 45. I understand from counsel that the Missouri Supreme Court has defined compactness to mean “closely united territory,” a concept which includes but is not limited to the physical shape of a district. *Pearson v. Koster*, 367 S.W.3d 36, 48 (Mo. banc 2012). I was also informed by counsel that the Missouri Supreme Court permits some deviation from compactness to accommodate other recognized redistricting considerations, including population density, natural boundary lines, the boundaries of political subdivisions (counties, municipalities, and precincts), the historical boundary lines of prior redistricting maps, and compliance with federal law, including the Voting Rights Act. *Id.* at 50, 53.

9. For this case, I was asked to determine whether CD4 and/or CD5 could have been drawn in a more compact manner, while keeping the remaining six Congressional districts identical to those in the Missouri FIRST Map, and if so, whether compliance with recognized redistricting considerations explains why CD4 and CD5 under the Missouri FIRST Map are not as compact as they could be.

10. As part of my analysis, I was asked by counsel to examine the properties of the CD4–CD5 boundary in the Missouri FIRST Map with respect to:

a. its effect on the preservation of political subdivisions, such as counties and cities, including but not limited to Jackson County and Kansas City;

b. the compactness of the resulting two districts;

c. the two districts' preservation of prior Congressional districts as defined by their historical boundary lines, preservation of state senate districts as defined by their current boundary lines, and allocation of Black Voting Age Population (BVAP).

I was also asked to provide my opinion on whether these properties are unusual or extreme compared to other CD4–CD5 boundary lines that might have been drawn, and whether these properties are affected by any attempt to avoid placing the current incumbents of CD4 and CD5 together in the same district.

11. To conduct this analysis, I used computer software to algorithmically generate an ensemble of 100,000 alternative maps, which vary the boundary between CD4 and CD5 while leaving the other six districts in the Missouri FIRST Map unchanged. The algorithm that generates these maps uses geographic and total-population data to construct geographically admissible (e.g., contiguous) districting plans that are nearly equipopulous within a small specified tolerance. Since the algorithm does not incorporate any demographic or partisan data in constructing the maps, it is race-blind and partisan-blind. Next, I measured each of these ensemble maps with respect to the properties listed in the previous paragraph—preservation of political subunits, compactness, and other considerations such as preservation of historical boundaries and allocation of BVAP—to establish a baseline for what we might expect from alternative maps that might have been drawn. Finally, I took those same measurements of the Missouri FIRST Map and compared them to those of the ensemble to see where the Missouri FIRST Map sits in the spectrum of possible maps—and in particular, whether it resembles a typical map or is an outlier with respect to any or all of the properties listed above. Further methodological details on this ensemble analysis are provided in Section IV.

12. Since my analysis is restricted to the portion of the state covered by CD4 and CD5, geographic regions mentioned in this report should be understood to mean the portion of those regions in CD4 or CD5, excluding portions in other districts, unless specified otherwise. For example, I may simply refer to "Kansas City" to mean "the portion of Kansas City lying within CD4 or CD5," excluding the part of Kansas City that is assigned to CD6 under the Missouri FIRST Map.

13. I am compensated at the rate of $300 per hour. My compensation does not depend in any way on the results of my analyses, the opinions I provide, or the outcome of this case.

III. Summary of Opinions

14. The Missouri FIRST Map splits the population of Jackson County between CD4 and CD5 more severely than 99.79% of ensemble maps. The ensemble maps tend to draw the vast majority of the Jackson County population (excluding the portion in CD6) into a single district, while the Missouri FIRST Map cracks it nearly in half. Furthermore, even if we total the largest intact portions of *all* county populations between CD4 and CD5, without singling out Jackson County, the Missouri FIRST Map is just as extreme.

15. The Missouri FIRST Map splits the population of Kansas City between CD4 and CD5 more severely than 99.59% of ensemble maps. The ensemble maps tend to draw the vast majority of the Kansas City population (excluding the portion in CD6) into one of these districts, while the Missouri FIRST Map cracks it into two large pieces (in addition to the third large piece in CD6). Furthermore, even if we total the largest intact portions of *all* municipality populations between CD4 and CD5, without singling out Kansas City, the Missouri FIRST Map is still more extreme than 98.95% of the ensemble maps.

16. The Missouri FIRST Map splits Voting Districts ("VTDs") between CD4 and CD5 more than 100% of the ensemble maps, both by number of splits and by total population of the largest VTD pieces; the ensemble maps only split VTDs to the minimum extent needed to ensure contiguous districts. While this is partly explained by the nonzero population-balance tolerance of the ensemble maps, the ensemble also includes maps with perfect population balance that do not split *any* VTDs between CD4 and CD5, indicating that the Missouri FIRST Map splits significantly more VTDs than necessary. VTDs are official Census geographic units that "include the wide variety of small polling areas, such as election districts, precincts, or wards, that State and local governments create for the purpose of administering elections,"[3] and they are among the primary units used in redistricting.

17. Across a range of compactness measures, the Missouri FIRST Map is consistently less compact than the ensemble maps, in many cases to an extreme degree. The CD4–CD5 boundary in the Missouri FIRST Map is longer than over 97.6% of the ensemble maps, and more than twice as long as the ensemble median, indicating that CD4 and CD5 are significant outliers on compactness, even after considering natural boundaries and population density. On every one of 9 standard district-compactness metrics, the Missouri FIRST Map has a worse average CD4–CD5 compactness score than 80% or more of the ensemble maps—in several cases worse than 90% or even 99%. On 7 of the 9 metrics, *both* the CD4 and CD5 scores, not just their average, are worse in the Missouri FIRST Map than the respective ensemble medians. Taken together with the previous observations about the severe splitting of Jackson County and Kansas City, this reflects that the Missouri FIRST Map avoids drawing a compact district based around Jackson County and Kansas City to a degree that is highly unusual in the ensemble.

[3] United States. Bureau of the Census. (1994). Geographic areas reference manual. US Department of Commerce, Economics and Statistics Administration, Bureau of the Census, at 14-1.

18. These shortcomings of the Missouri FIRST Map cannot be explained by preservation of previous Congressional districts or current state senate districts in CD4 and CD5. By the same methodology used to measure splitting of counties and municipalities, these other regions are shown to be poorly preserved by the Missouri FIRST Map compared to the ensemble.

19. These shortcomings of the Missouri FIRST Map also cannot be explained by any attempt to increase minority voting strength or to create a minority district required by the Voting Rights Act. BVAP is more cracked between CD4 and CD5 in the Missouri FIRST Map than over 97% of ensemble maps, which reflects a significantly lower-than-typical BVAP percentage in CD5 under the Missouri FIRST Map. Again, the ensemble maps used for comparison are drawn by a completely race-blind algorithm.

20. These findings do not change if one constrains the ensemble maps to keep the current CD4 and CD5 incumbents in separate districts, or to have a stricter population-equality tolerance. While the exact numbers vary slightly, the conclusions are the same.

IV. Methodology

21. Although there is sophisticated mathematics and computational science under the hood, the basic idea behind ensemble analysis is quite simple. Given a particular districting map (such as the Missouri FIRST Map), we would like to know how its properties compare to the other maps that could have been drawn. Since it is impossible to compare to every possible map, of which there are an astronomical number, we use computer algorithms to randomly generate a large sample of such maps, referred to as an *ensemble*. Then, when we measure a property of the Missouri FIRST Map, we can see where it ranks among the measurements of the ensemble maps: Does it rank somewhere in the middle, resembling a typical ensemble map, or is it among a small

percentage of ensemble maps to one extreme? For some properties, one would want an enacted map to be not just typical, but *better* than typical. In particular, if Missouri were adhering to the compactness requirement, we would expect the Missouri FIRST Map to resemble maps of *above*-average compactness in the ensemble.

A. Construction of Ensemble Maps

22. The first step in constructing an ensemble is specifying the geographic units on which the ensemble maps are to be based, where each district is to be composed of whole, unsplit units. The most basic units are 2020 Census Blocks, which are the smallest geographic units on which the Census reports data. For redistricting, one often works with larger units called VTDs, each of which consists of a collection of whole Blocks. Missouri contains 253,632 Blocks and 4,604 VTDs. Since the ensemble maps only involve changes to CD4 and CD5, I based them on geographic units constructed as follows:

a. Starting with all 2020 Census Blocks in Missouri, I selected the Blocks assigned to CD4 or CD5 in the Missouri FIRST Map. There are 67,517 of these.

b. I then combined Blocks belonging to the same contiguous piece of a VTD. (Since VTDs or portions thereof in CD4 and CD5 are not always contiguous, this step ensures that all districts in the ensemble are contiguous.) This results in 1,190 units consisting of the contiguous VTD pieces in CD4 or CD5. Block-level data, such as population, were aggregated to these units accordingly.

This construction is illustrated in Figure 1 below.

Figure 1: An illustration of the geography on which the ensemble maps are constructed. *Left*: The Missouri FIRST Map, with CD4 and CD5 highlighted. *Center*: We limit the area being redrawn to the territory covered by CD4 and CD5. *Right*: This area is divided into 1,190 units, corresponding to contiguous pieces of VTDs. The ensemble maps assign each of these units to a district without splitting them. Note that the VTDs are smaller and more numerous in densely-populated areas, particularly the Kansas City area in the northwest corner of the map.

23. The ensemble was generated using a widely used, widely studied, and widely accepted Markov Chain Monte Carlo ("MCMC") algorithm called *ReCom*[4] (for "recombination"). MCMC algorithms are similar in spirit to card shuffling: to randomly sample from the huge number of possible ways to order a deck of cards, we can simply start with any initial ordering and shuffle repeatedly. The ReCom algorithm "shuffles" a districting plan by randomly picking two adjacent districts, combining them, and then randomly splitting the combined region into two districts again, while ensuring population balance within a small specified tolerance. This step is repeated over and over again, and the ensemble is the collection of districting maps obtained in the resulting sequence.

24. Since we have restricted attention to just two districts, the ReCom algorithm always recombines and splits the same pair of districts. This makes the algorithm much simpler to understand: At each step, it randomly splits the combined region covered by CD4 and CD5.

[4] DeFord, D., Duchin, M., & Solomon, J. (2021). Recombination: A Family of Markov Chains for Redistricting. *Harvard Data Science Review*, 3(1).

I employed a "region-aware" version of ReCom that performs the random splitting in a way that avoids splitting counties.

25. The ensemble maps were constructed to satisfy the following constraints:

a. Both districts are geographically contiguous.

b. Each district's population deviates from the ideal district population of 769,364 by no more than ±1%. (Results for an additional ensemble with an even stricter population tolerance of ±0.1% are reported in Appendix 3, illustrating that minor population deviations do not significantly impact the ensemble results.)

26. Other than the preference for maps that avoid splitting counties, mentioned above, I did not introduce any artificial parameters or constraints that would cause the ensemble to prefer or require certain types of maps to others. In particular, no compactness conditions were imposed to steer the algorithm toward compact maps and away from less-compact ones.

27. The ensemble-construction algorithm uses only geographic and total-population data, so it is blind to other factors; in particular, it is partisan-blind and race-blind.

B. Description of Ensemble Analysis Software

28. I performed the ensemble analysis in this report using the open-source software library *GerryChain*[5], together with the companion software library *GerryTools*[6], which implement the core computational algorithms (e.g., ReCom) used to construct the ensembles.

29. I also wrote additional code to process the raw Census data and Block-assignment data for the Missouri FIRST Map into the formats needed for *GerryChain* and *GerryTools*—including aggregating Census Blocks into the geographic units described in paragraph 22 above—as well as code to organize and score the Missouri FIRST Map and ensemble maps, and

[5] https://github.com/mggg/GerryChain, accessed December 20, 2025
[6] https://github.com/mggg/gerrytools, accessed December 20, 2025

to generate the tables and figures that appear in this report. This code was written in the programming language Python using open-source libraries, primarily the *Pandas* and *GeoPandas* libraries for computing with geographic data.

30. All computational analysis in this report was conducted on my personal computer.

C. Sources of Data

31. The following data were used to conduct the analysis described in this report:

a. 2020 Census TIGER/Line shapefiles, containing geographic data;

b. 2020 Census Redistricting Data (P.L. 94-171) Summary Files, containing population and demographic data;

c. Block-assignment files for VTDs, Census "Places" (including municipalities), the two preceding Congressional districting maps (116th and 118th Congress, corresponding to the 2012 and 2022 maps), and 2022 Missouri Senate districts;

d. Block-assignment files for the Missouri FIRST Map, i.e., 2025 H.B. 1 Congressional districts.

e. Census Block IDs for the CD4 and CD5 incumbents.

Items a, b, and c were downloaded directly from the United States Census. Item d was downloaded directly from the Missouri Spatial Data Information Service[7], as linked from the Missouri Office of Administration Redistricting Office[8]. Item e was obtained from the Census Geocoder service, based on home addresses provided to me by counsel.

32. For purposes of BVAP analysis, "Black" means "any part Black," which is commonly used as a metric for redistricting and can be calculated using official Census data. In

[7] https://data-msdis.opendata.arcgis.com/search?tags=hb1, accessed December 20, 2025
[8] https://budplan.oa.mo.gov/redistricting-office/2025-us-congressional-house-maps, accessed December 20, 2025

each Census Block, I computed BVAP by summing the 2020 VAP data columns corresponding to Black alone or in combination with other races from the P.L. 94-171 summary file.

V. Ensemble Analyses

A. Splitting of Counties and Municipalities Between CD4 and CD5

33. The CD4–CD5 boundary in the Missouri FIRST Map splits exactly one county, Jackson County. Likewise, none of the ensemble maps splits more than one county.

34. However, the *number* of splits does not tell us about the *severity* of the splitting. Some maps split off only a small portion of a county's population, leaving the vast majority of county residents together in the same district. Other maps split the county's population nearly evenly between CD4 and CD5, cracking the county in two. The latter is far more problematic from the perspective of maintaining the closely united territory that comprises a county and its residents. Cracking communities this way also dilutes the political power of residents by making *both* pieces (even the larger of the two) too small for the community to form an effective bloc in *either* district. This can be quantified by counting the population of the largest piece of each county between CD4 and CD5, i.e., how many people live in the same district as the majority of their fellow county residents. (Here and henceforth, when I refer to the "largest piece" of a county or other region, I mean the piece with the larger population between CD4 and CD5—as opposed to the largest by area or some other measurement.)

35. Table 1, below, shows that Jackson County is extremely cracked in the Missouri FIRST Map—more cracked than 99.79% of the ensemble maps. In the ensemble, the median population of the largest Jackson County piece is 701,167, which is the full population of the portion of Jackson County within these two districts. This means that the majority (in fact, about 56%) of ensemble maps do not split this portion of Jackson County at all. By contrast, in the

Missouri FIRST Map, the largest piece has a population of only 370,868; the population of Jackson County within these two districts is split about 47%–53% between CD4 and CD5, respectively. As the "Percentile" column indicates, only 0.21% of the ensemble maps keep less of this population intact between CD4 and CD5 than the State's map—meaning that the Missouri FIRST Map is more severely cracked than the remaining 99.79% of ensemble maps.

Table 1: County splitting between CD4 and CD5. The Missouri FIRST Map splits county population more severely than 99.79% of ensemble maps.

	Ensemble Median	Missouri FIRST Map	Percentile
Counties Split	1	1	50.35%
Population of Largest Jackson County Piece	701,167	370,868	0.21%
Total Population of Largest County Pieces	1,500,170	1,208,429	0.21%

36. Furthermore, this cracking of Jackson County *cannot* be explained as a trade-off for less severe county splitting overall at the expense of a single unlucky county. To test this, I totaled the largest-population pieces of *all* counties between CD4 and CD5, without singling out Jackson County or any other county for special focus. As shown in Table 1, the ensemble maps have a median total population of 1,500,170 across the largest county pieces. The combined population of CD4 and CD5 is 1,538,728—meaning that, in a typical ensemble map, about 97.5% of this population lives in the largest piece of their county. By contrast, in the Missouri FIRST Map, the total population across the largest county pieces is only 1,208,429—meaning that only about 78.5% of the population lives in the largest piece of their county. Just as when we considered Jackson County alone, the Missouri FIRST Map cracks counties in CD4 and CD5 overall more severely than all but 99.79% of the ensemble maps, with only the remaining 0.21% scoring lower (as shown in the "Percentile" column). Far from a trade-off, the extreme splitting of Jackson County drives the overall population numbers to be just as extreme.

37. The severe cracking of county population in the Missouri FIRST Map, compared with the ensemble maps, is visually striking in Figures 2 and 3 below. These plots, known as *histograms*, show the values observed in the 100,000 ensemble maps as blue bars. Each bar corresponds to a range of values on the horizontal axis—here, population values—and the height of each bar indicates what percentage of the ensemble maps have values in that range. For example, the rightmost bar in Figure 2 shows that about 60% of the ensemble maps have a Jackson County piece whose population is in the range of values between the left and right edges of the bar (from around 675,000 to the maximum of 701,167). The ensemble median is shown as a green line, and the Missouri FIRST Map value is shown as a yellow line. Both figures show the yellow line far to the left of the blue bars, indicating that it is extremely rare to observe a value this low in the ensemble.

Figure 2: Histogram showing the largest portion of Jackson County population left intact between CD4 and CD5. The Missouri FIRST Map (yellow line) is an extreme outlier compared to the ensemble maps (blue bars, median shown as green line).

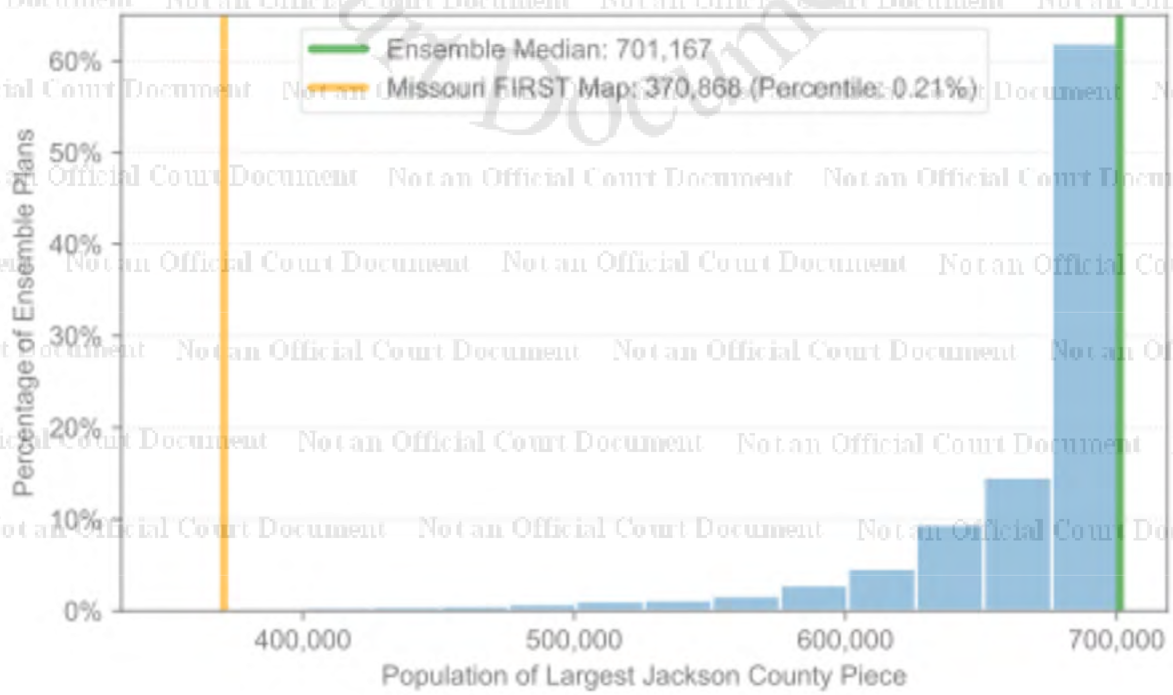
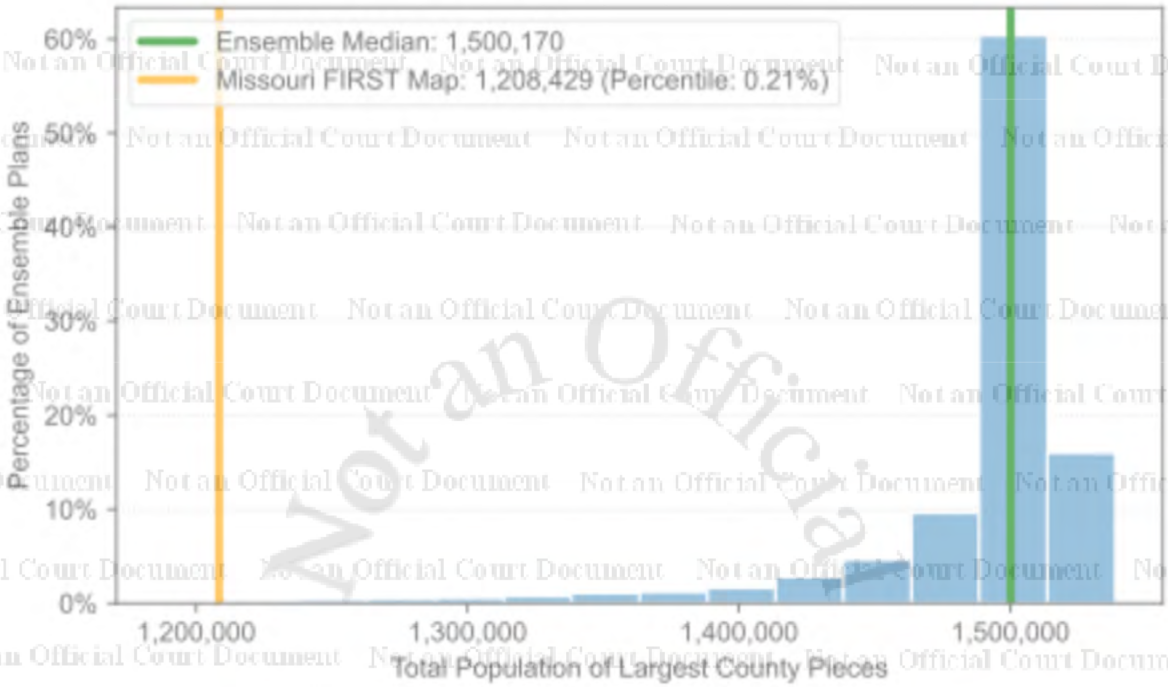


Figure 3: Histogram showing the population of the largest county pieces between CD4 and CD5, totaled over all counties.



38. Table 2, below, summarizes the results of a similar splitting analysis for municipalities[9]. The Missouri FIRST Map splits a somewhat higher-than-average number of municipalities (six) between CD4 and CD5, compared to the ensemble (four), but the *severity* of these splits—as measured by population—is far more extreme than the mere number of splits.

Table 2: Municipality splitting between CD4 and CD5. The Missouri FIRST Map splits the population of Kansas City in these two districts more severely than 99.59% of ensemble maps, and the population of municipalities overall more severely than 98.95% of ensemble maps.

	Ensemble Median	Missouri FIRST Map	Percentile
Municipalities Split	4	6	78.42%
Population of Largest Kansas City Piece	300,523	174,515	0.41%
Total Population of Largest Municipality Pieces	1,075,857	960,864	1.05%

[9] Here, a "municipality" is a Census Place whose Legal Statistical Area Description (LSAD) code indicates that it is a city, town, or village, not counting unincorporated Census Designated Places (CDPs).

39. Kansas City is more cracked between CD4 and CD5 in the Missouri FIRST Map than 99.59% of the ensemble maps; only the remaining 0.41% of the ensemble maps score lower. (This is even without accounting for the Missouri FIRST Map cracking Kansas City across a third district, CD6, which contains a comparably-sized piece of the city and is outside the scope of this analysis.) In the ensemble, the median population of the largest Kansas City piece is 300,523, which is the full population of the portion of Kansas City within these two districts. This means that the majority (in fact, about 64%) of ensemble maps have this portion of Kansas City entirely contained within a single district. By contrast, in the Missouri FIRST Map, the larger of the two pieces of Kansas City between CD4 and CD5 has a population of only 174,515, giving a roughly 42%–58% split of the Kansas City population within these two districts.

40. This cracking of Kansas City, like that of Jackson County, cannot be explained as a trade-off to achieve less severe splitting of municipalities overall; to the contrary, the extreme splitting of Kansas City drives the overall municipality-splitting numbers to be extreme as well. As shown in Table 2, the ensemble maps have a median total population of 1,075,857 across the largest pieces of all municipalities. The population of CD4 and CD5 living in municipalities is 1,087,487—meaning that a typical ensemble map places about 98.9% of this population in the largest piece of their municipality. By contrast, in the Missouri FIRST Map, the total population across the largest municipality pieces is 960,864—meaning that only about 88.4% of the municipal population lives in the largest piece of their municipality. Municipalities are more cracked between CD4 and CD5 in the Missouri FIRST Map than 98.95% of ensemble maps; only the remaining 1.05% keep less of this population intact.

41. The histograms in Figures 4 and 5, below, illustrate this severe cracking of Kansas City specifically, and of municipalities overall, between CD4 and CD5. Just as with the county-

splitting histograms, the yellow line representing the Missouri FIRST Map is far to the left of the blue bars, showing that a value this low is rarely observed among the ensemble maps.

Figure 4: Histogram showing the largest portion of Kansas City population left intact between CD4 and CD5.

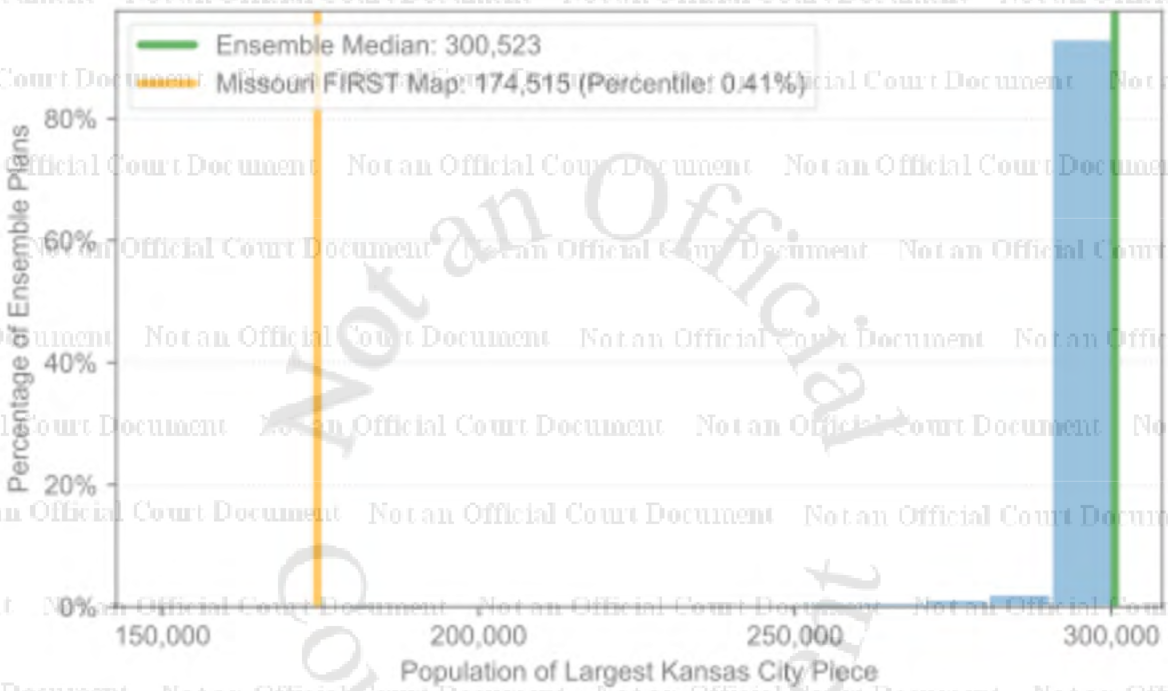
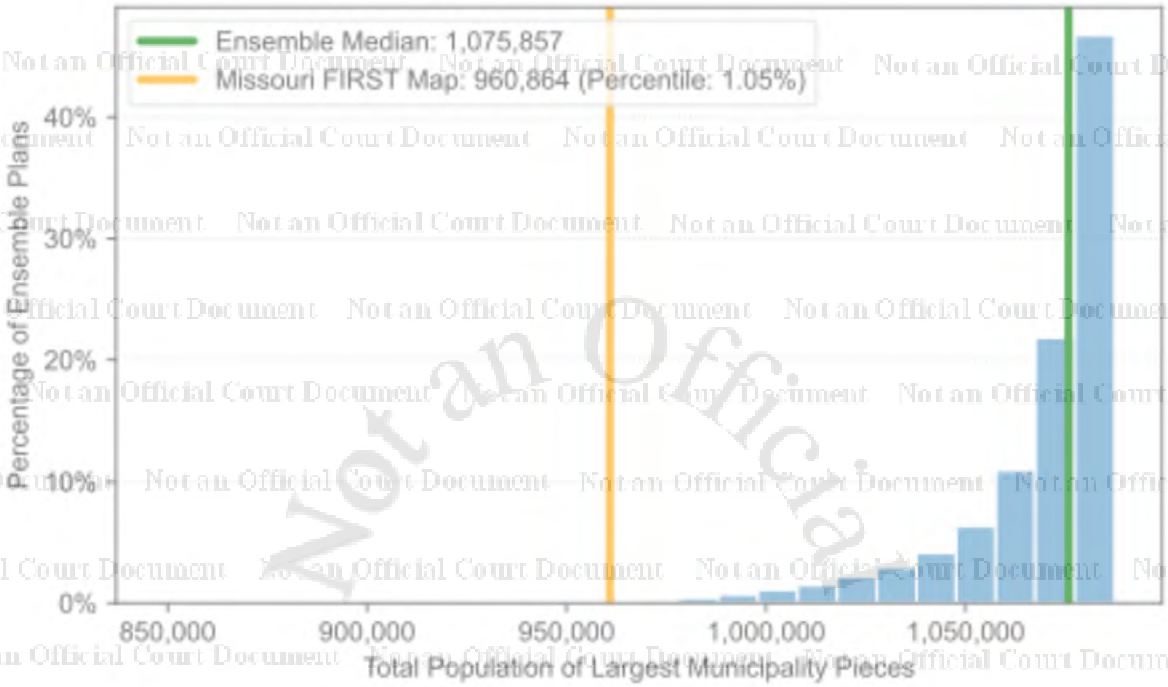


Figure 5: Histogram showing the population of the largest municipality pieces between CD4 and CD5, totaled over all municipalities.



42. The Missouri FIRST Map splits VTDs between CD4 and CD5 more than 100% of the ensemble maps, both by number of splits and by total population of the largest VTD pieces, as shown below in Table 3 and in Figures 6 and 7. As previously described in paragraph 22, the ensemble maps only split VTDs to the minimum extent needed to ensure contiguous districts.

Table 3: VTD splitting between CD4 and CD5.

	Ensemble Median	Missouri First Map	Percentile
VTDs Split	0	18	100.00%
Total Population of Largest VTD Pieces	1,538,728	1,534,476	0.00%

Figure 6: Histogram showing the number of VTDs split between CD4 and CD5.

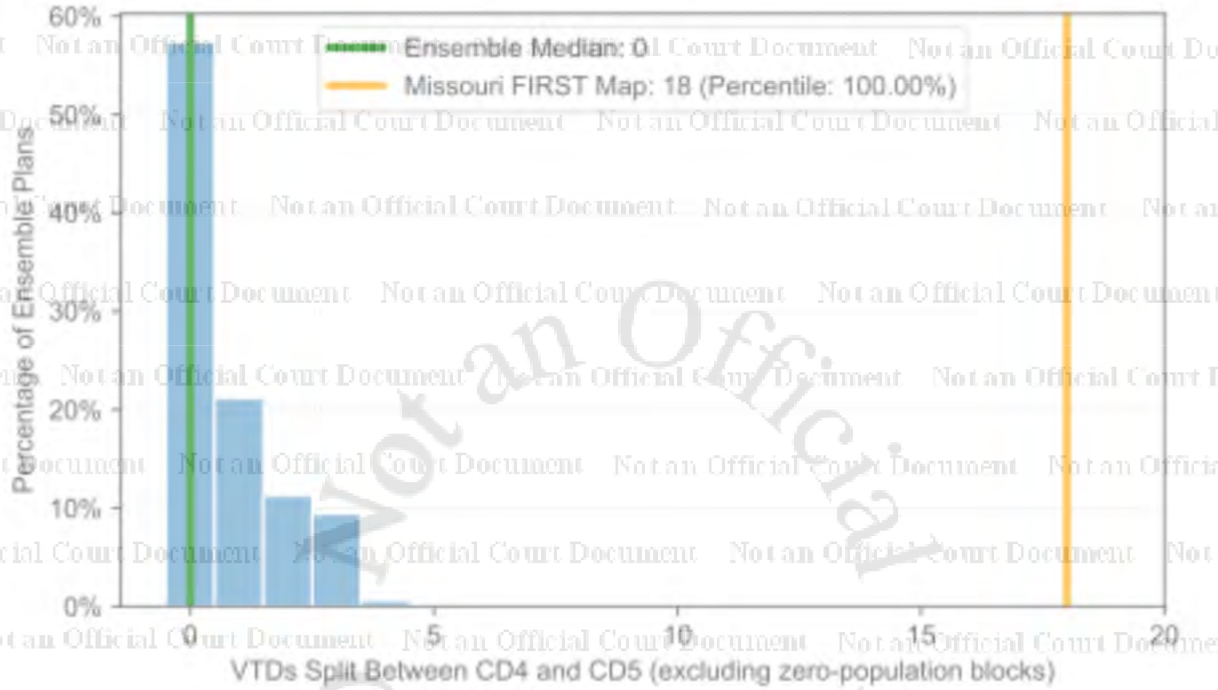
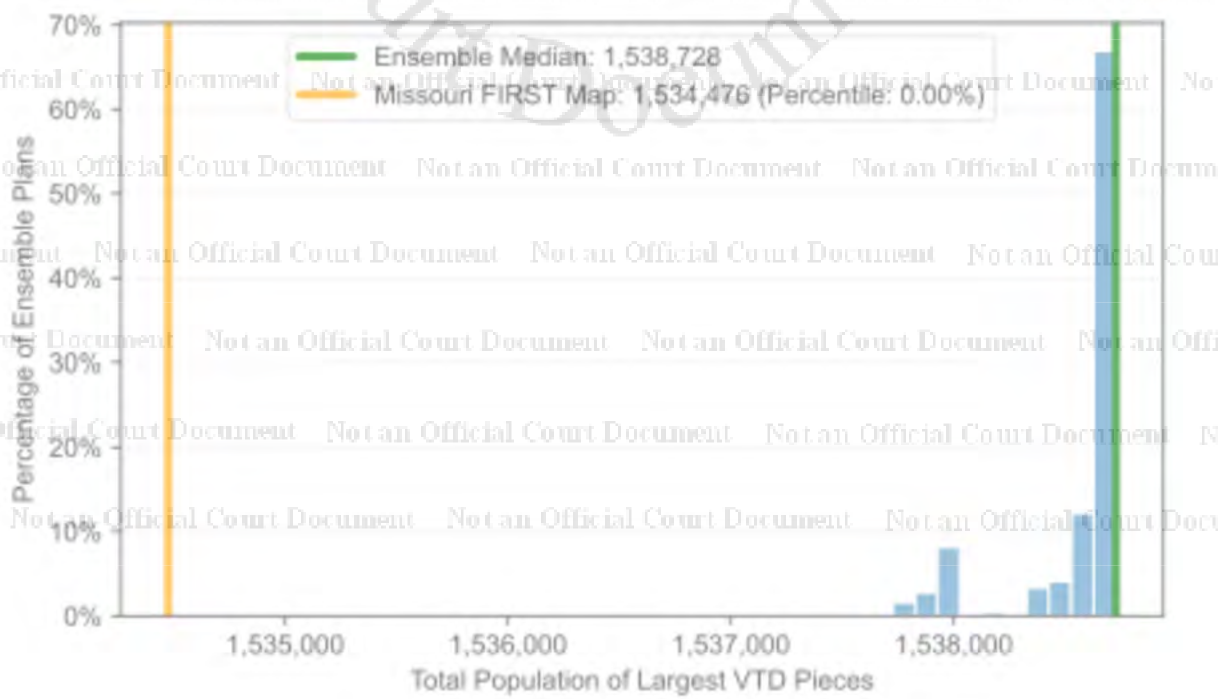


Figure 7: Histogram showing the population of the largest VTD pieces between CD4 and CD5, totaled over all VTDs.



43. To be sure, this is partly because the ensemble maps allow a ±1% tolerance in population: for any particular ensemble map, one would need to make small Block-level adjustments in order to achieve perfect population balance, and this adjustment might introduce VTD splits. However, this ensemble, together with the ±0.1% population tolerance ensemble discussed in Appendix 3, includes maps with perfect population balance that do not split *any* VTDs between CD4 and CD5. Examples of these maps are shown in Appendix 4.

B. Compactness Metrics

44. There are numerous metrics for quantifying "compactness" of districts and districting maps—but across a wide variety of standard metrics, the Missouri FIRST Map stands out as remarkably and consistently less compact in CD4 and CD5 compared to the ensemble.

45. One hallmark of compact plans is that they tend to have relatively short boundaries. A circle has the shortest boundary among all shapes enclosing a given area and is often considered an ideal of compactness, while—on the other hand—long, snaking boundaries have been a traditional signature of gerrymandering. Since the outer boundary of the CD4–CD5 region is held fixed in the ensemble maps, the only boundary line that changes between maps is the interior boundary separating CD4 from CD5.

46. There are two main ways to measure the size of the CD4–CD5 boundary. The first, and simplest, is to measure its length in miles. A second, subtler approach is to count what are called "cut edges" between Census Blocks. The terminology comes from mathematics—but in plain English, a "cut edge" corresponds to a pair of adjacent Blocks that lie on opposite sides of the boundary. The number of cut edges counts how many adjacent pairs of Blocks are divided by the boundary, one placed into CD4 and its neighbor into CD5. There are important interpretive benefits of looking at cut edges:

a. Many compactness metrics are based on the notion that there is an "ideal shape" (such as a circle or square) for all districts, regardless of constraints imposed by natural geography, political subunits, and how population is distributed. Of course, this is not true: a twisting, winding boundary could be carving up population in a very unnatural way, or it could be innocuously following a natural boundary, like a river, that has an irregular geometric shape. Census Blocks typically follow these natural boundaries, so the cut edges metric incorporates these geographic and population factors that other metrics omit.

b. Boundary length in miles does not account for population density. A mile of boundary can separate many more people in a densely-populated city than it can in a sparsely-populated area. For example, changing a relatively short segment of the district boundary in Kansas City can have a greater effect on the district's population than a much longer segment in a rural county. The cut edges metric accounts for this, since Census Blocks are more numerous and denser in areas of denser population (as previously shown in Figure 1 for VTDs).

47. However we measure the CD4–CD5 boundary—whether by length in miles or cut edges—the Missouri FIRST Map scores as much less compact than the ensemble maps. Indeed, it both stretches halfway across the state *and* carves through densely populated areas. As Table 4 shows, below, the CD4–CD5 boundary is longer in miles than 97.69% of the ensemble maps, and over twice as long as the ensemble median. Likewise, the Missouri FIRST Map cuts more edges along the CD4–CD5 boundary than 97.63% of the ensemble maps, and over twice as many edges as the ensemble median. This is also apparent from the histograms in Figures 8 and 9, below.

Table 4: Size of the boundary between CD4 and CD5, as measured by length in miles and by cut edges. On both metrics, the Missouri FIRST Map is less compact than over 97.6% of ensemble maps, having a CD4–CD5 boundary over twice the size of the ensemble median.

	Ensemble Median	Missouri FIRST Map	Percentile
Length in Miles	130.17	273.37	97.69%
Cut Edges	333	792	97.63%

Figure 8: Histogram showing the length in miles of the CD4–CD5 boundary.

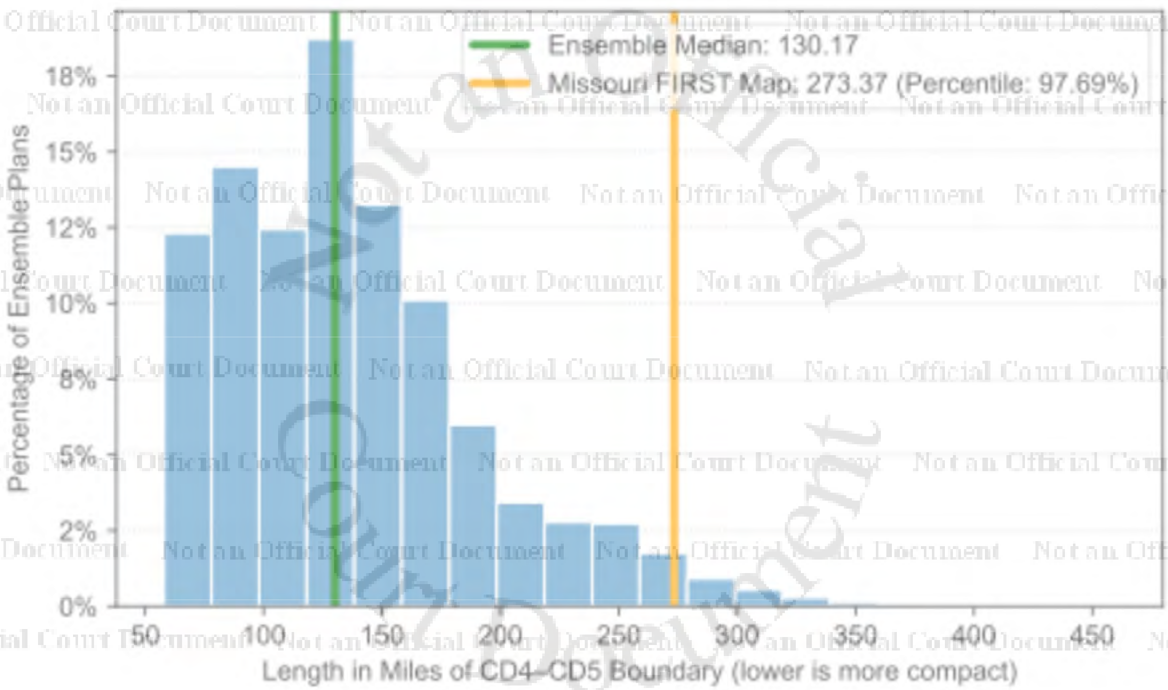
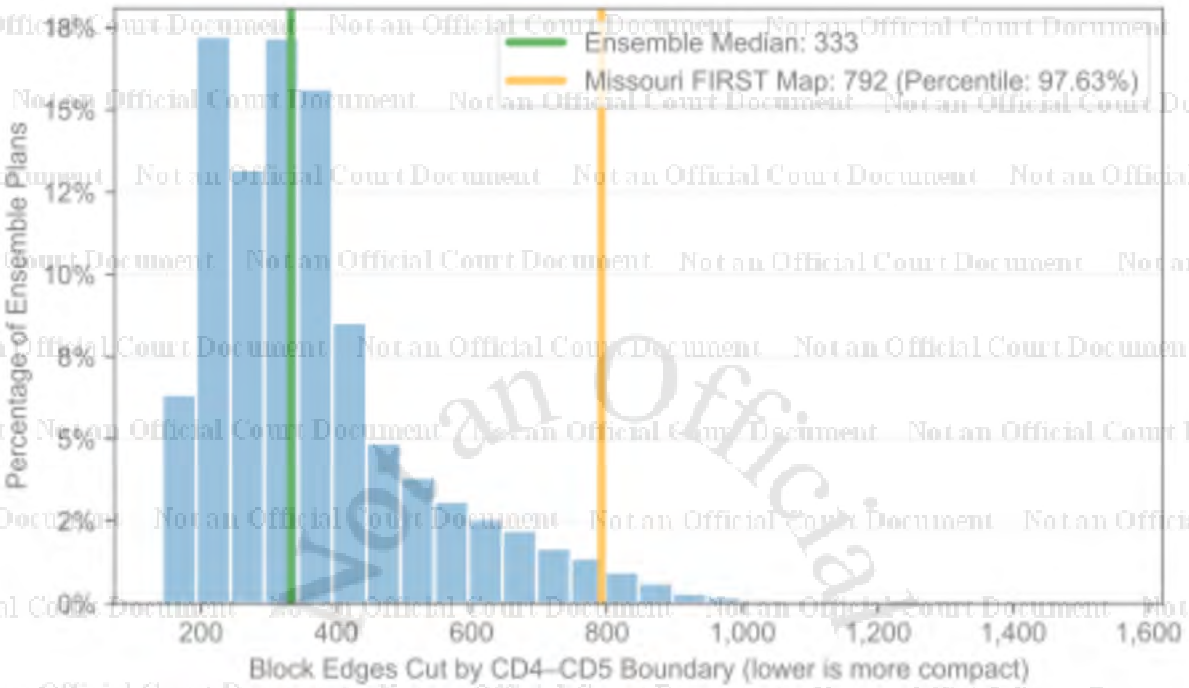


Figure 9: Histogram showing the number of cut edges along the CD4–CD5 boundary.



48. In addition to boundary length and cut edges, we consider 9 standard metrics for measuring the compactness of individual districts. With these metrics, each district gets its own compactness score, so we will have two numbers for each map rather than just one. However, since the districts vary from map to map in the ensemble, it is not necessarily meaningful to say which score corresponds to "CD4" and which corresponds to "CD5." Instead, for each map, we will consider the *minimum* of the two scores, the *maximum* of the two scores, and the *average* of the two scores. The 9 metrics are:

a. Reock,

b. (Alternate) Schwartzberg,

c. Polsby–Popper,

d. Population Polygon,

e. Area/Convex Hull,

f. Population Circle,

g. Ehrenburg,

h. Perimeter,

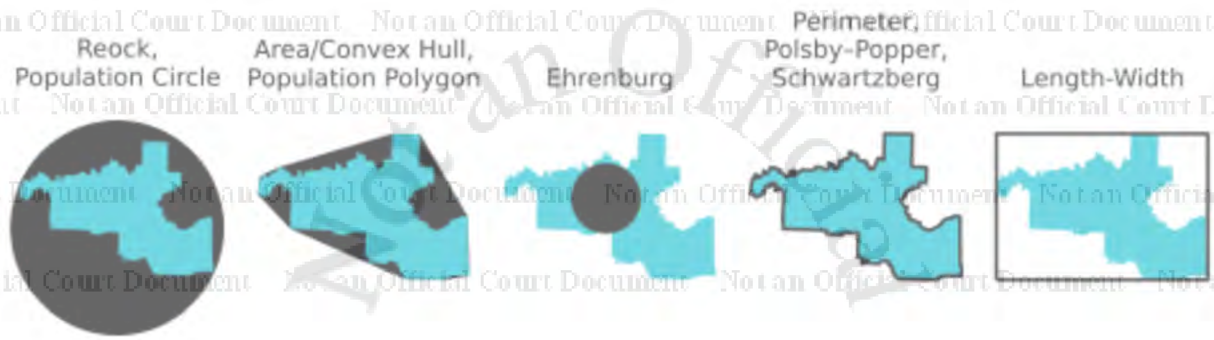
i. Length-Width.

This is the full list of district-by-district compactness metrics reported in a September 10, 2025, memo on the Missouri FIRST Map[10], showing scores computed using the software *Maptitude*. Figure 10 contains a visual guide to these scores and a brief description of the idea behind each.[11]

49. Note that *Maptitude* distinguishes between two versions of the Schwartzberg score, which it calls "Schwartzberg" and "Alternate Schwartzberg." The original metric proposed by Schwartzberg in 1966 used a rough approximation to district perimeter, since it was not feasible at the time to compute perimeter precisely.[12] The Schwartzberg metric using the true perimeter is what *Maptitude* calls "Alternate Schwartzberg," and this is the version I compute.

[10] Adam Kincaid, *Memo to Representative Dirk Deaton, HB 1 Sponsor, re: The Missouri First Map* (Sept. 10, 2025).
[11] For these 9 metrics, minor variations in the scores may arise from differences in the underlying map projection or in the software used to calculate those scores. Those variations, to the extent they exist, do not affect the conclusions I present here.
[12] Duchin, M. (2022). Explainer: Compactness, by the Numbers. In *Political Geometry*, eds. Duchin, M., and Walch, O., Birkhäuser/Springer, Cham, at p. 30.

Figure 10: A visual guide to the 9 standard district-by-district compactness metrics, illustrated with CD5 of the Missouri FIRST Map shown in blue. *From left to right*: Reock compares the area of the district to that of the smallest circle enclosing it; Population Circle compares these shapes' populations rather than areas. Area/Convex Hull compares the area of the district to that of its "convex hull," which contains all straight-line paths between points in the district; Population Polygon compares these shapes' populations rather than areas. Ehrenburg compares the area of the district to that of the largest circle contained inside it. Polsby–Popper compares the area of the district to that of the circle with the same perimeter; Schwartzberg compares the perimeter of the district to that of the circle with the same area. Finally, Length-Width simply measures the difference between the length and width of a rectangle enclosing the district.



50. For some of these compactness scores, a higher score means better compactness; for others, a lower score means better compactness. To avoid confusion, I will present the ensemble analysis of all the higher-is-better scores first, followed by the lower-is-better scores.

51. Table 5, below, shows the results of the ensemble analysis for the 6 higher-is-better compactness scores, where the three rows for each score correspond to the minimum, average, and maximum score between the two districts. The percentile column is shaded red whenever the Missouri FIRST Map is less compact than the ensemble median and green whenever it is more compact. Only one cell of 18 is shaded green; in the remaining 17 red cells, the percentiles range from 0.13% to 27.01%, meaning that the Missouri FIRST Map is less compact than 72.99% of ensemble maps at best, and less compact than 99.87% of them at worst. For *all six* metrics, the average score of CD4 and CD5 is less compact in the Missouri FIRST Map than 81.35% of ensemble maps at best, and less compact than 99.82% of them at worst.

Table 5: Compactness metrics for which higher scores indicate greater compactness.

		Ensemble Median	Missouri FIRST Map	Percentile
Reock	min	0.39	0.35	27.01%
	avg	0.47	0.39	1.39%
	max	0.56	0.43	0.50%
Polsby–Popper	min	0.30	0.20	5.53%
	avg	0.32	0.27	6.04%
	max	0.36	0.33	24.51%
Population Polygon	min	0.62	0.60	22.72%
	avg	0.77	0.62	0.25%
	max	0.92	0.63	0.13%
Area/Convex Hull	min	0.78	0.70	16.80%
	avg	0.81	0.76	18.65%
	max	0.84	0.82	25.27%
Population Circle	min	0.29	0.35	92.23%
	avg	0.53	0.38	0.18%
	max	0.76	0.40	0.47%
Ehrenburg	min	0.35	0.25	15.65%
	avg	0.43	0.37	13.20%
	max	0.52	0.49	19.94%

52. On several of these measures, the Missouri FIRST Map ranks as extremely non-compact by comparison to the ensemble maps. In particular:

a. On Reock, the most compact district in the Missouri FIRST Map (CD4) scores worse than the most compact district in 99.50% of ensemble maps. The average Reock compactness of CD4 and CD5 is worse than 98.61% of ensemble maps.

b. On Population Polygon, the most compact district in the Missouri FIRST Map (CD4) scores worse than the most compact district in 99.87% of ensemble maps, and the average score is worse than 99.75% of ensemble maps.

c. On Population Circle, the most compact district in the Missouri FIRST Map (CD5) scores worse than the most compact district in 99.53% of ensemble maps, and the average score is worse than 99.82% of the ensemble maps.

53. The lone green cell reflects a trade-off between moderately better Population Circle compactness in CD4 at the expense of vastly worse compactness in CD5 and on average. Specifically, the Population Circle compactness score for the Missouri FIRST Map is about 20% better than the ensemble median in one district, but the ensemble median is about *90%* better in the other. This poor trade-off has an intuitive explanation: For maps where CD5 is a compact district based around Jackson County and Kansas City, the small circle around CD5 will contain very little of the CD4 population, but the huge circle around CD4 will contain all of the CD5 population and more.

54. On Polsby–Popper, the least compact district in the Missouri FIRST Map, CD5, scores worse than 94.47% of ensemble maps, and the average score is worse than 93.96% of ensemble maps. (The most compact district, CD4, is "only" worse than 75.49% of ensemble maps.) This is well below the vast majority of ensemble maps, albeit not as extreme an outlier as some of the other compactness scores.

55. Table 6, below, shows the results of the ensemble analysis for the three lower-is-better compactness scores. As above, the percentile column is shaded red whenever the Missouri FIRST Map is less compact than the ensemble median, and green whenever it is more compact. Again, all but one of the cells is red, corresponding to worse compactness of CD4 and CD5 in the Missouri FIRST Map than 75.50% to 99.35% of the ensemble maps. For all three metrics, the average compactness of CD4 and CD5 is worse than 89.34% or more of ensemble maps.

Table 6: Compactness metrics for which lower scores indicate greater compactness.

		Ensemble Median	Missouri FIRST Map	Percentile
(Alternate) Schwartzberg	min	1.67	1.74	75.50%
	avg	1.76	1.98	94.68%
	max	1.83	2.22	94.48%
Perimeter in Miles	min	242.09	628.75	99.35%
	avg	529.76	672.96	97.69%
	max	805.30	717.16	3.63%
Length-Width in Miles	min	0.33	11.80	92.77%
	avg	12.28	30.88	89.34%
	max	24.23	49.95	78.04%

56. On perimeter, the more compact district in the Missouri FIRST Map (CD4) is worse than 99.35% of the ensemble maps, and the average perimeter is worse than 97.69% of ensemble maps. (This is the same 97.69% previously encountered for the length of the CD4–CD5 boundary.) The lone green cell again reflects a trade-off, where one district's perimeter is shorter in the Missouri FIRST Map, at the expense of a much longer perimeter in the other district and on average. Specifically, in the Missouri FIRST Map, one district has a perimeter about 88 miles shorter than the corresponding ensemble median, but the other has a perimeter about *387 miles longer* than the corresponding ensemble median. Again, this poor trade-off has an intuitive explanation: A compact CD5 based around Jackson County and Kansas City would have a much shorter perimeter than either district in the Missouri FIRST Map, while the corresponding CD4 would have a longer perimeter.

57. On (Alternate) Schwartzberg, the least compact district in the Missouri FIRST Map, CD5, scores worse than 94.48% of ensemble maps, and the average score is worse than 94.68% of ensemble maps. (The most compact district, CD4, is "only" worse than 75.50% of ensemble maps.) This is essentially what we observed for Polsby–Popper, due to a mathematical relationship between the Schwartzberg and Polsby–Popper scores.

58. Histograms visualizing the CD4–CD5-average compactness scores for these 9 metrics are included in Appendix 1.

C. Splitting of Previous Congressional Districts and State Senate Districts Between CD4 and CD5

59. To analyze how the Missouri FIRST Map preserves or splits previous Congressional districts and current state senate districts between CD4 and CD5, I employed the same methodology used above to analyze splitting of counties, municipalities, and VTDs. As before, only the portions of a state senate district that are within CD4 or CD5 of the Missouri FIRST Map are considered. I analyze the preservation of previous Congressional districts, because, as mentioned above, I understand historical boundary lines to be a relevant districting consideration articulated by the Missouri Supreme Court. I also analyze the current state senate districts, because the Kincaid memo, *see supra* note 9, identified adherence to state senate district boundary lines as a potential justification for the Missouri FIRST Map.

60. Table 7, below, shows that the Missouri FIRST Map splits the population of 2012 Congressional districts between CD4 and CD5 more severely than 98.71% of ensemble maps, with only the remaining 1.29% of the ensemble maps preserving less population (as indicated in the "Percentile" column). For 2022 Congressional districts, the splitting is more severe than 98.85% of ensemble maps, with only the remaining 1.15% maps preserving less population. The histograms in Figures 11–14, below, illustrate the number and severity of these splits.

Table 7: Splitting of 2012 and 2022 Congressional districts between CD4 and CD5.

	Ensemble Median	Missouri First Map	Percentile
2012 Congressional Districts Split	2	4	99.75%
Total Population of Largest District Pieces	1,412,696	1,007,069	1.29%
2022 Congressional Districts Split	2	3	82.76%
Total Population of Largest District Pieces	1,374,550	1,006,066	1.15%

Figure 11: Histogram showing the number of 2012 Congressional districts split between CD4 and CD5.

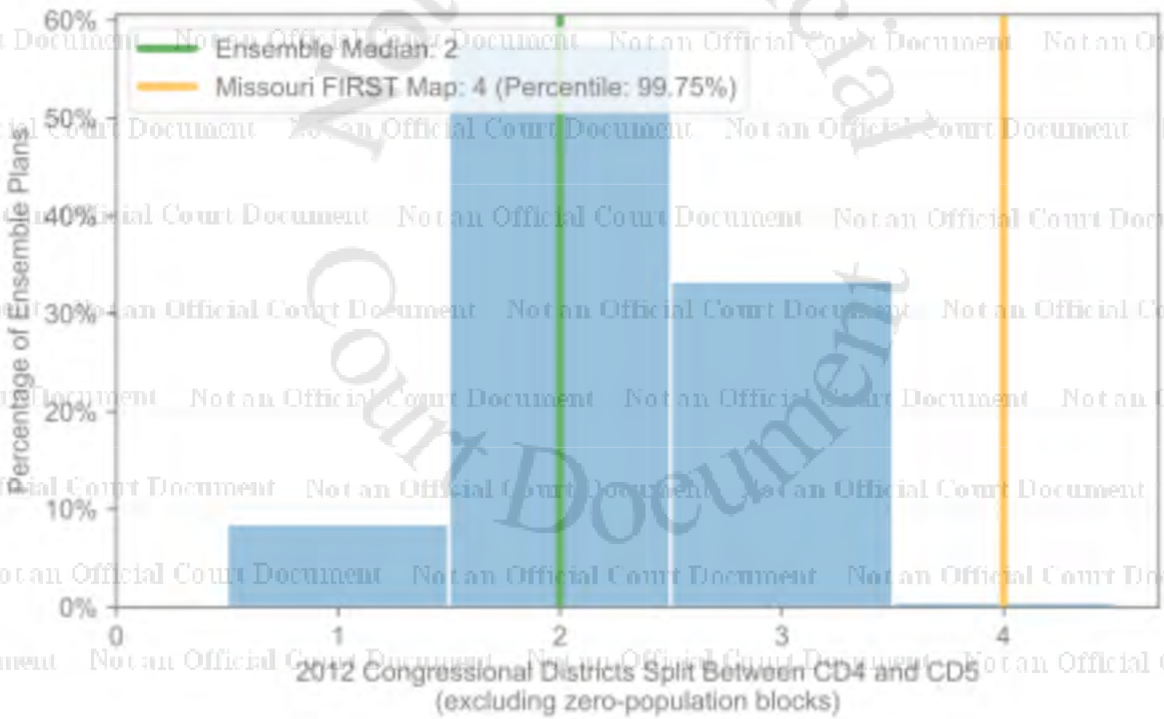


Figure 12: Histogram showing the population of the largest 2012 Congressional district pieces between CD4 and CD5, totaled over all districts.

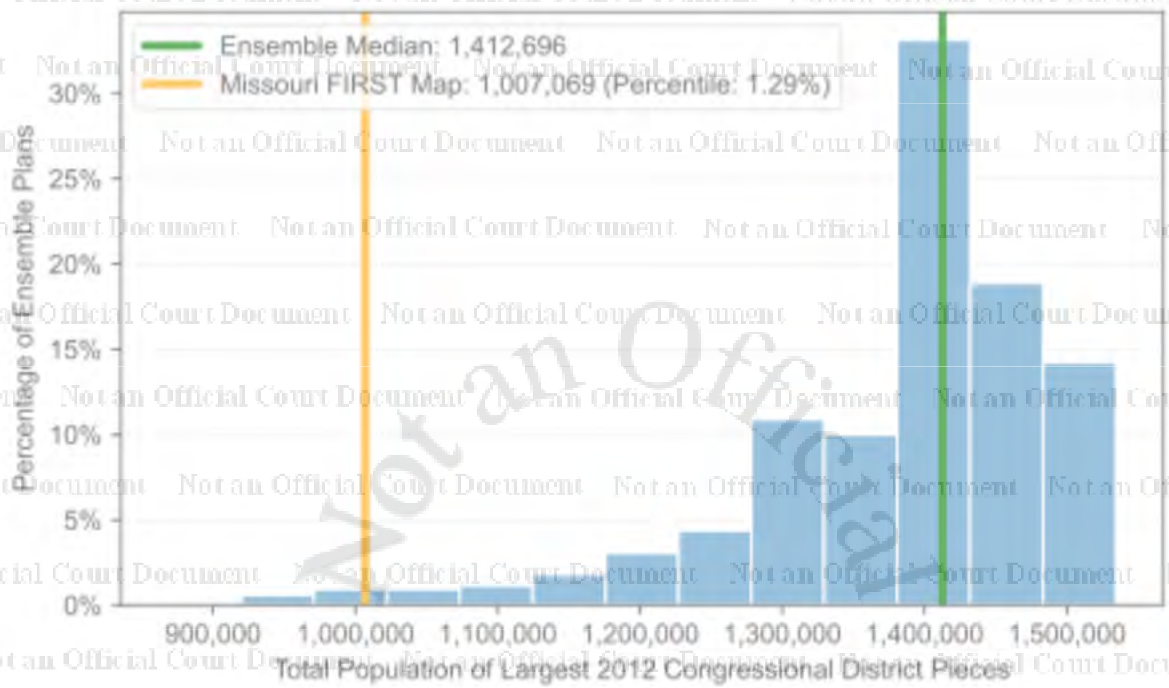


Figure 13: Histogram showing the number of 2022 Congressional districts split between CD4 and CD5.

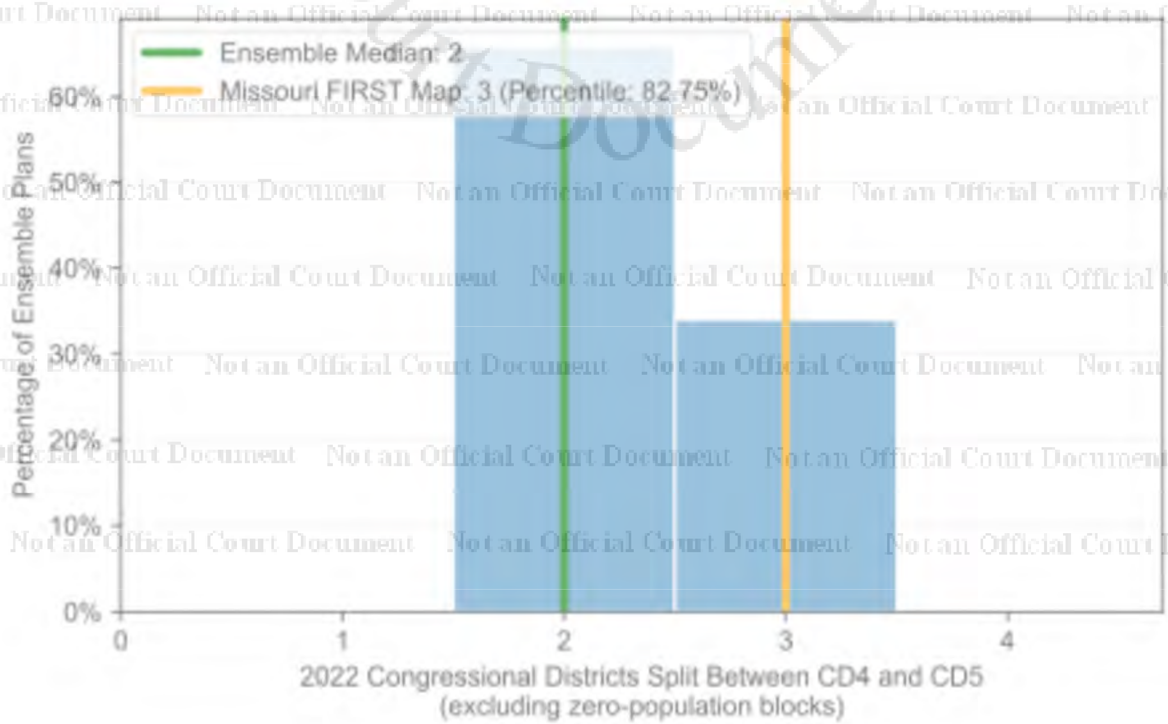
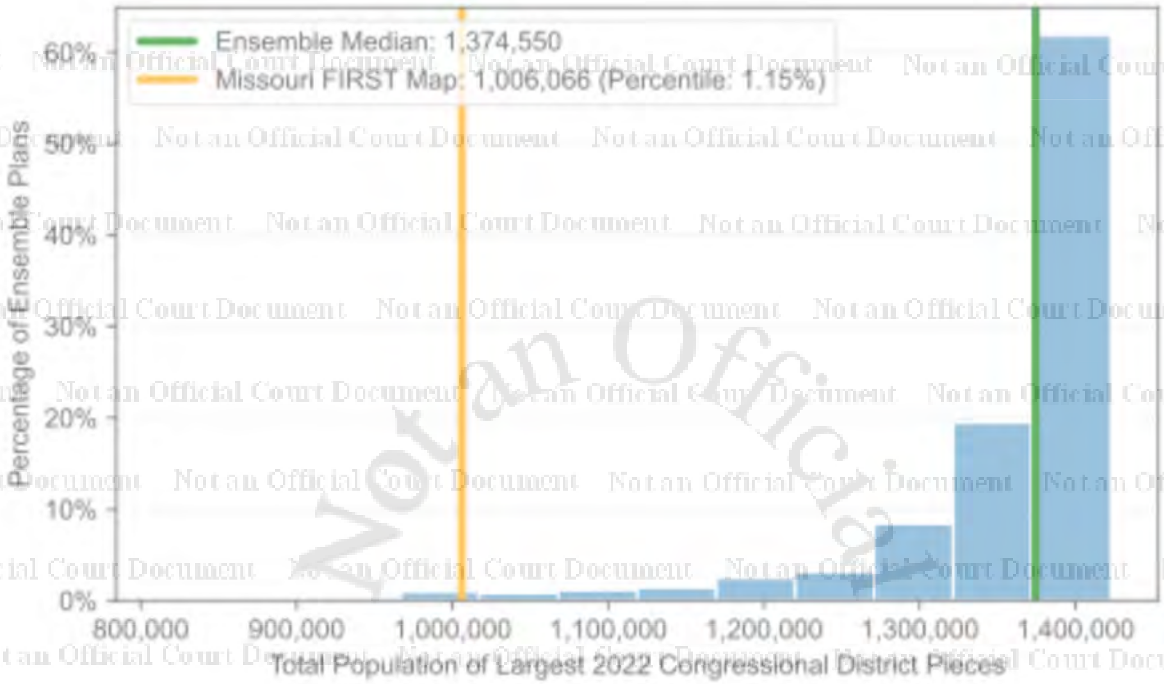


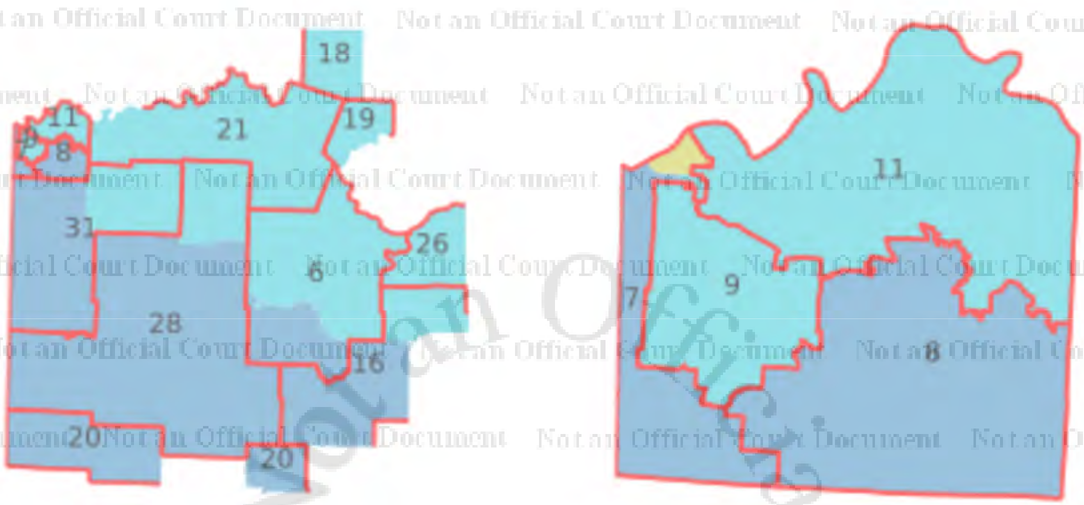
Figure 14: Histogram showing the population of the largest 2022 Congressional district pieces between CD4 and CD5, totaled over all districts.



61. Next, we analyze the splitting of state senate districts between CD4 and CD5 of the Missouri FIRST Map. Because the Kincaid memo indicated that "align[ing] closely with the existing Missouri Senate Map" was considered in "the split of Jackson County," *id.* at p. 4, I examine whether this stated priority to respect state senate districts is supported by the data.

62. For context, Figure 15 below overlays the current state senate district ("SD") lines and numbers over CD4 and CD5 of the Missouri FIRST Map, as well as a detailed view of Jackson County (including the portion in CD6). In addition to the splitting between CD4 and CD5 of SD6, SD16, SD28, and SD31 clearly visible at left, the Jackson County detail at right shows a *three-way* split of SD7 between CD4, CD5, and CD6. Other senate districts are seen to be split with neighboring Congressional districts in the Missouri FIRST Map, which in particular leaves two disconnected pieces of SD20 in CD4.

Figure 15: The Missouri FIRST Map compared with state senate district boundaries (in red) and district numbers. *Left*: Detail of the area covered by CD4 and CD5. *Right*: Detail of Jackson County, with portion of CD6 shown in green. Note the three-way split of SD7.



63. As Table 8, Figure 16, and Figure 17 show below, the Missouri FIRST Map splits state senate districts between CD4 and CD5 a greater number of times, and more severely, than about 90% of ensemble maps. While not as extreme an outlier as the splitting of Jackson County and Kansas City, the Missouri FIRST Map is still worse in this regard than the vast majority of alternative maps in the ensemble—and thus it does not resemble a map for which "align[ing] closely with the existing Missouri Senate Map" was indeed a priority.

Table 8: Splitting of Missouri Senate districts between CD4 and CD5.

	Ensemble Median	**Missouri First Map**	**Percentile**
State Senate Districts Split	2	5	93.11%
Total Population of Largest State Senate District Pieces	1,464,752	1,382,297	10.88%

Figure 16: Histogram showing the number of Missouri Senate districts split between CD4 and CD5.

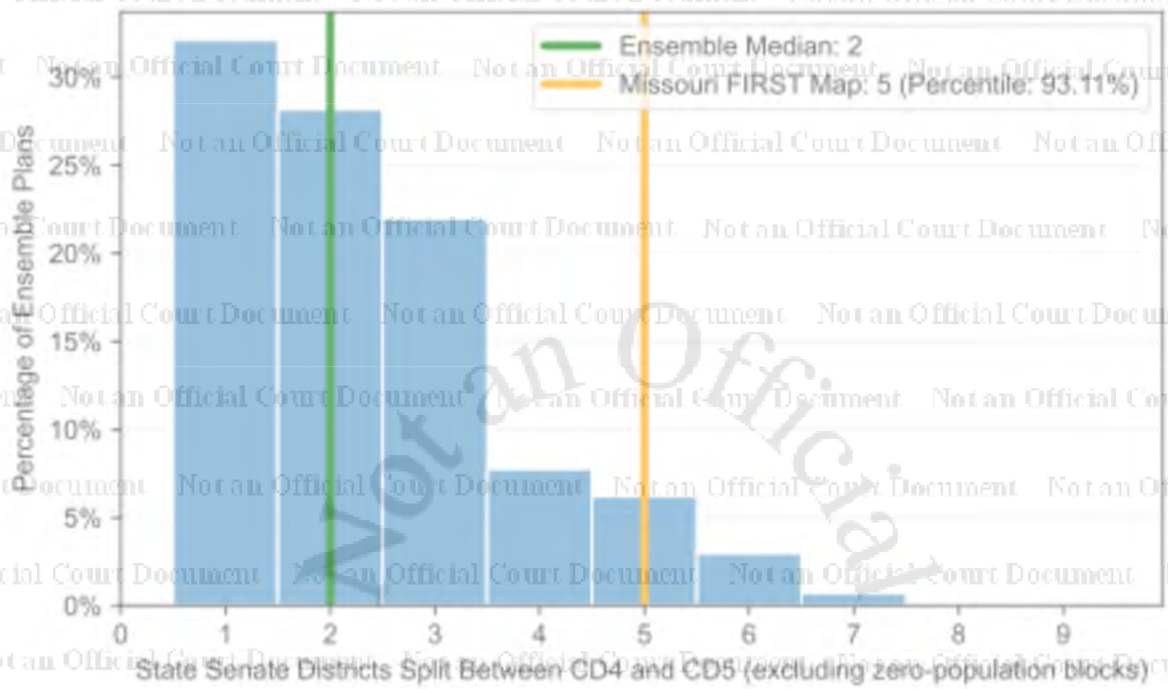
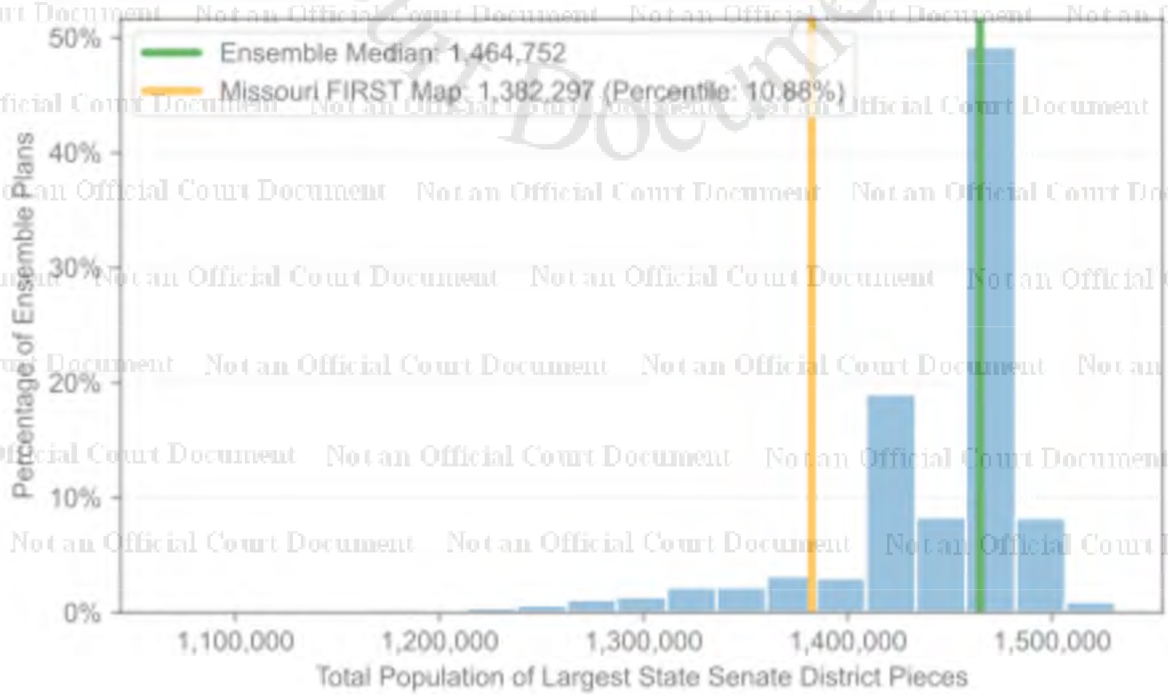


Figure 17: Histogram showing the population of the largest Missouri Senate district pieces between CD4 and CD5, totaled over all districts.



D. Allocation of Black Voting Age Population

64. Black voters make up a minority of CD4 and CD5, both before and after the enactment of the Missouri FIRST Map. The BVAP for CD4 went from 5.01% to 7.64% of VAP under the Missouri FIRST Map, and the BVAP for CD5 dropped from 22.45% to 17.97%. For that reason and those discussed below, compliance with federal law, such as any requirement to create a majority-Black district under the Voting Rights Act, does not explain why CD4 and CD5 in the Missouri FIRST Map are non-compact.

65. Notably, BVAP is more cracked between CD4 and CD5 of the Missouri FIRST Map than 97.31% of the ensemble maps, as shown in Table 8, below. (Note that the percentiles in the table add to 100%, because the sum of BVAP between the two districts is the same in every map, i.e.: the total BVAP in the area covered by the two districts.)

Table 9: BVAP in the lower-BVAP district (CD4 in the Missouri FIRST Map) and higher-BVAP district (CD5 in the Missouri FIRST Map). BVAP is significantly lower in the higher-BVAP district and higher in the lower-BVAP district compared to the ensemble plans.

	Ensemble Median	Missouri FIRST Map	Percentile
BVAP: Lower of CD4 and CD5	25,444	45,836	97.31%
BVAP: Higher of CD4 and CD5	126,168	105,776	2.69%

66. These results show that the Missouri FIRST Map's distribution of Black voters is an extreme outlier compared to the ensemble maps, which were created without employing any racial data or preference. Whereas the ensemble median has 126,168 Black voting-age residents in the higher-BVAP district, the Missouri FIRST Map has only 105,776 Black voting-age residents.

67. The histograms in Figures 18 and **19** clearly illustrate this lower-than-typical BVAP in the higher-BVAP district, which is CD5 in the Missouri FIRST Map, compared with the

ensemble maps. The usual green line marks the ensemble median, while the yellow line indicates that the Missouri FIRST Map is a clear outlier.

Figure 18: Histogram showing BVAP in the larger-BVAP district between CD4 and CD5.

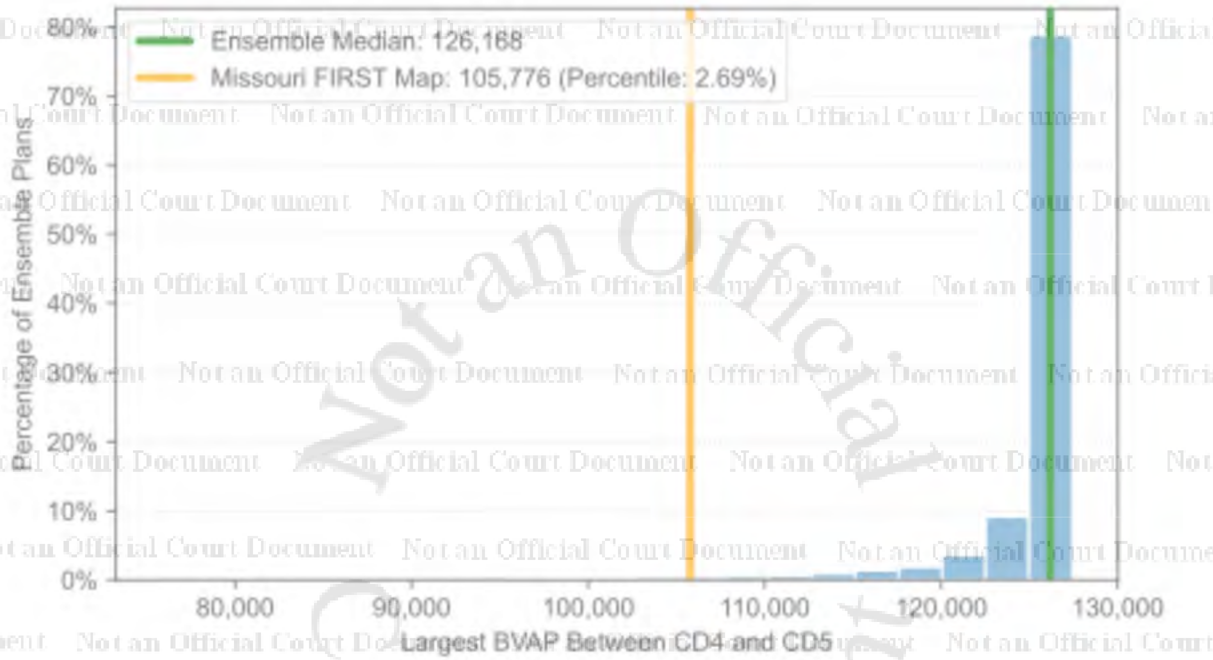
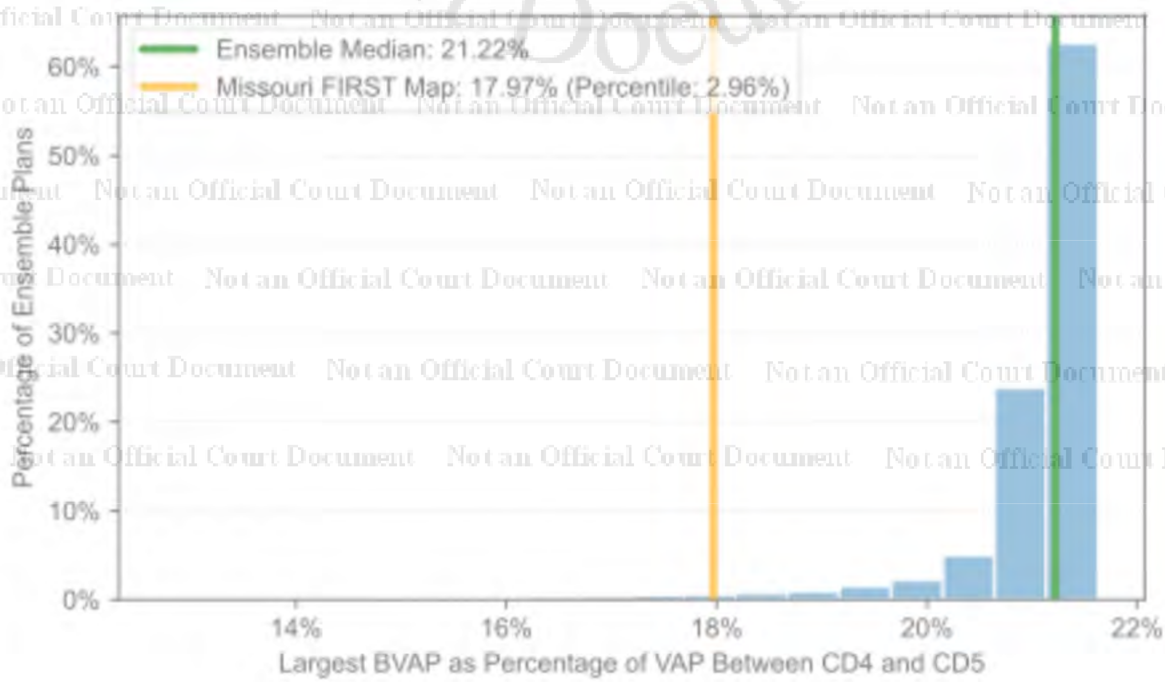


Figure 19: Histogram showing BVAP as a percentage of VAP in the larger-BVAP district between CD4 and CD5.



E. Effects of Incumbency Constraints and Population Equality on the Conclusions of the Preceding Analyses

68. The preceding analyses are based on the ensemble of 100,000 maps constructed as described in Section IV.A. This construction does not impose any constraints on the properties of the ensemble maps with respect to the current CD4 and CD5 incumbents, and it allows for these districts' populations to deviate by ±1% from perfect population balance. To determine whether each of these factors has an effect on the conclusions above, I conducted two additional sets of ensemble analyses: one using only the ensemble maps that place the current CD4 and CD5 incumbents into separate districts, and the other using a new ensemble of 100,000 maps that tightens the population tolerance to ±0.1% but otherwise employs the same methodology.

69. For these alternative ensembles, the exact numbers (scores, percentiles, etc.) vary slightly, but the top-line results and conclusions are unchanged from those above. I will briefly summarize these analyses here; numerical tables appear in Appendix 2 for the incumbency-constrained ensemble and in Appendix 3 for the stricter-population-balance ensemble.

70. To perform the incumbency-constrained ensemble analysis, I selected only those maps from the original ensemble placing the current CD4 and CD5 incumbents into separate districts, and discarded those that place them in the same district. This is no different than if one were to impose the constraint at the time of construction: the algorithm would simply discard any maps it drew that violated the constraint, which is the same as discarding them after the fact. About two-thirds of the original ensemble maps place the CD4 and CD5 incumbents in the same district, which is unsurprising since both reside in the greater Kansas City area; discarding these leaves an ensemble of 33,439 maps satisfying the incumbency constraint.

71. The results of the incumbency-constrained ensemble analysis, supported by the numerical tables in Appendix 2, are summarized as follows:

a. The Missouri FIRST Map cracks the population of Jackson County and counties overall more severely than about 99.9% of incumbency-constrained ensemble maps, and the population of Kansas City and cities overall more severely than over 99% of incumbency-constrained ensemble maps.

b. The Missouri FIRST Map splits more VTDs and cracks VTD population more severely than all 100% of the incumbency-constrained ensemble maps.

c. The Missouri FIRST Map CD4–CD5 boundary is longer than over 98% of the incumbency-constrained ensemble maps, whether measured in miles or cut edges.

d. On the compactness metrics where the Missouri FIRST Map previously appeared most extreme—less compact than over 99% of original ensemble maps—it is just as extreme or worse by comparison with the incumbency-constrained ensemble. Some of the metrics where the Missouri FIRST Map appeared only moderately non-compact have moved closer to the ensemble median. Two of the scores that were below the original ensemble median are very close to the median of the incumbency-constrained ensemble, and the corresponding table cells are colored yellow. As before, the remaining cells are colored red when they are below the ensemble median and green when they are above the ensemble median.

e. The Missouri FIRST Map cracks the population of previous Congressional districts more severely than over 99% of the incumbency-constrained ensemble, and the population of state senate districts more severely than over 90% of the incumbency-constrained ensemble.

f. The Missouri FIRST Map cracks BVAP more severely than 95.65% of the incumbency-constrained ensemble maps.

The top-line conclusions about the original ensemble therefore remain true for the ensemble of incumbency-constrained maps.

72. The stricter-population-balance ensemble analysis can be summarized much more briefly: The numerical tables in Appendix 3 are very close to those for the original ensemble, generally differing by only a small fraction. The top-line conclusions about the original ensemble therefore remain true for the alternative ensemble satisfying stronger population equality.

73. In short, neither incumbency protection nor a more stringent population equality threshold affects my conclusion that CD4 and CD5 under the Missouri FIRST Map are extremely non-compact in a manner that cannot be explained by recognized redistricting principles.

* * *

I declare under penalty of perjury that the foregoing is true and correct to the best of my knowledge. I reserve the right to revise, update, or supplement my opinions as new information becomes available to me.

Date: December 30, 2025

Dr. Ari J. Stern

APPENDIX 1: DISTRICT-AVERAGE COMPACTNESS HISTOGRAMS

Figure 20: Histogram showing the average Reock score between CD4 and CD5.

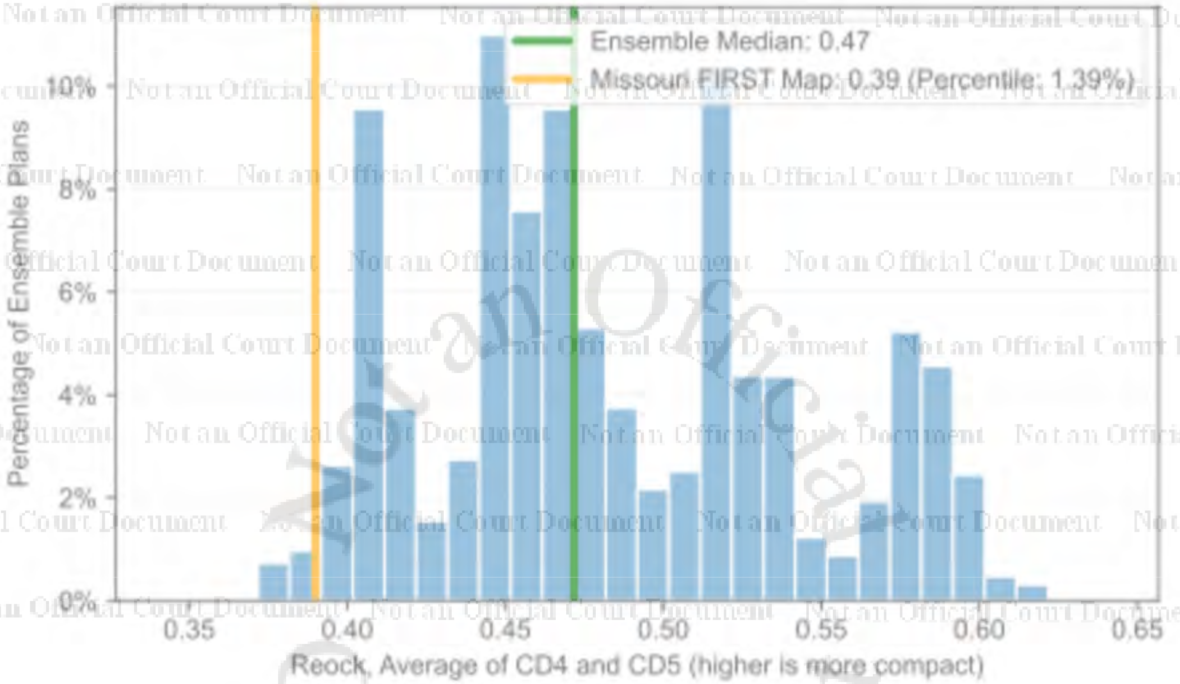


Figure 21: Histogram showing the average Polsby–Popper score between CD4 and CD5.

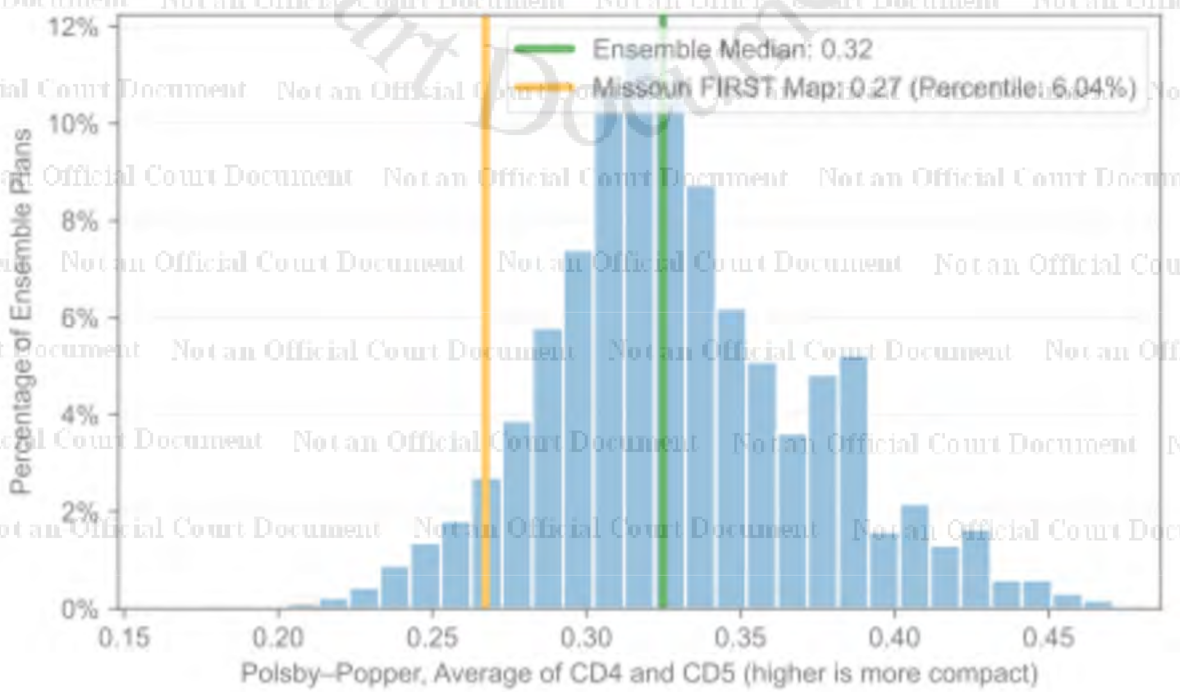


Figure 22: Histogram showing the average Population Polygon score between CD4 and CD5.

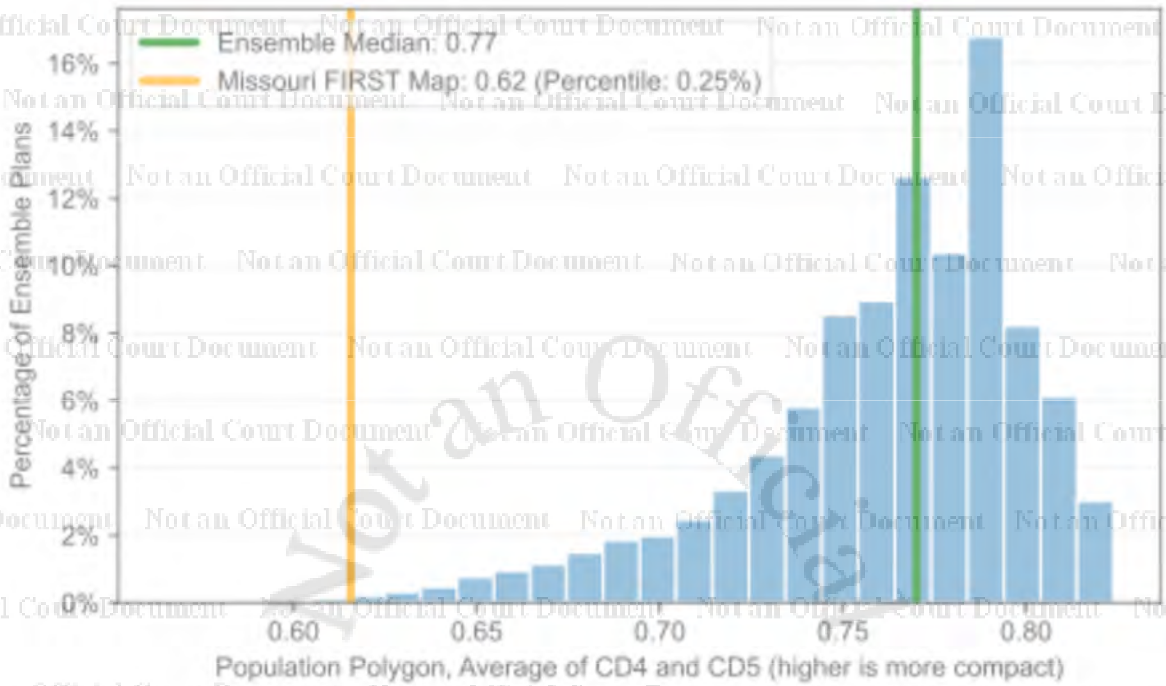


Figure 23: Histogram showing the average Area/Convex Hull score between CD4 and CD5.

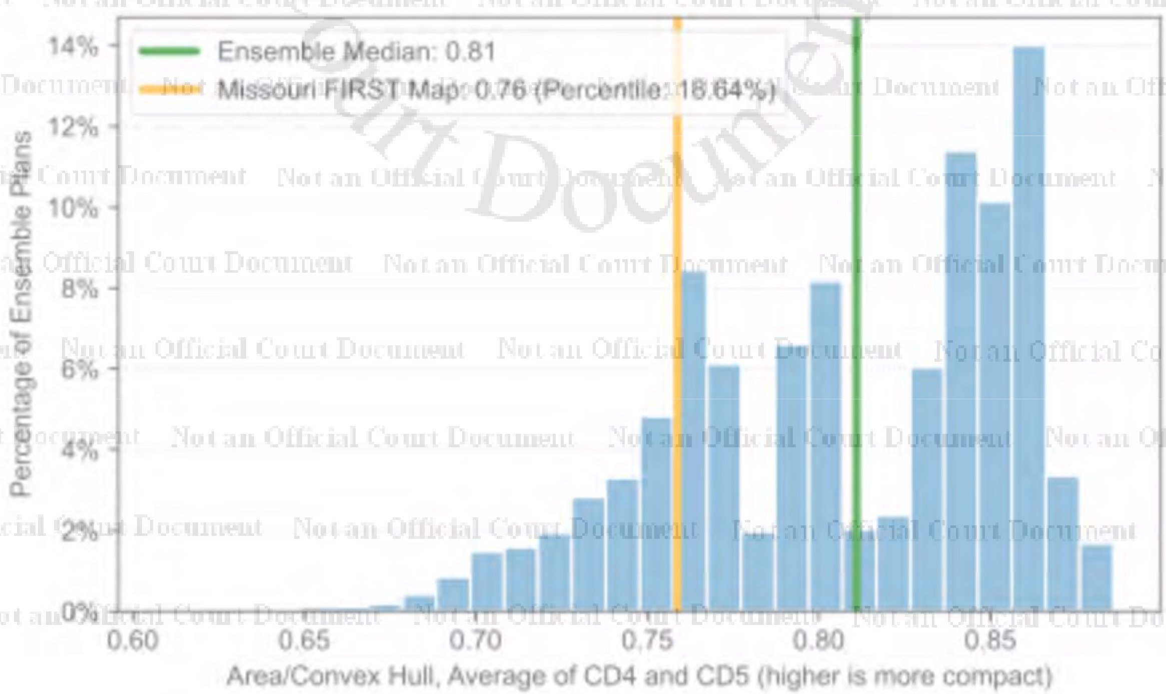


Figure 24: Histogram showing the average Population Circle score between CD4 and CD5.

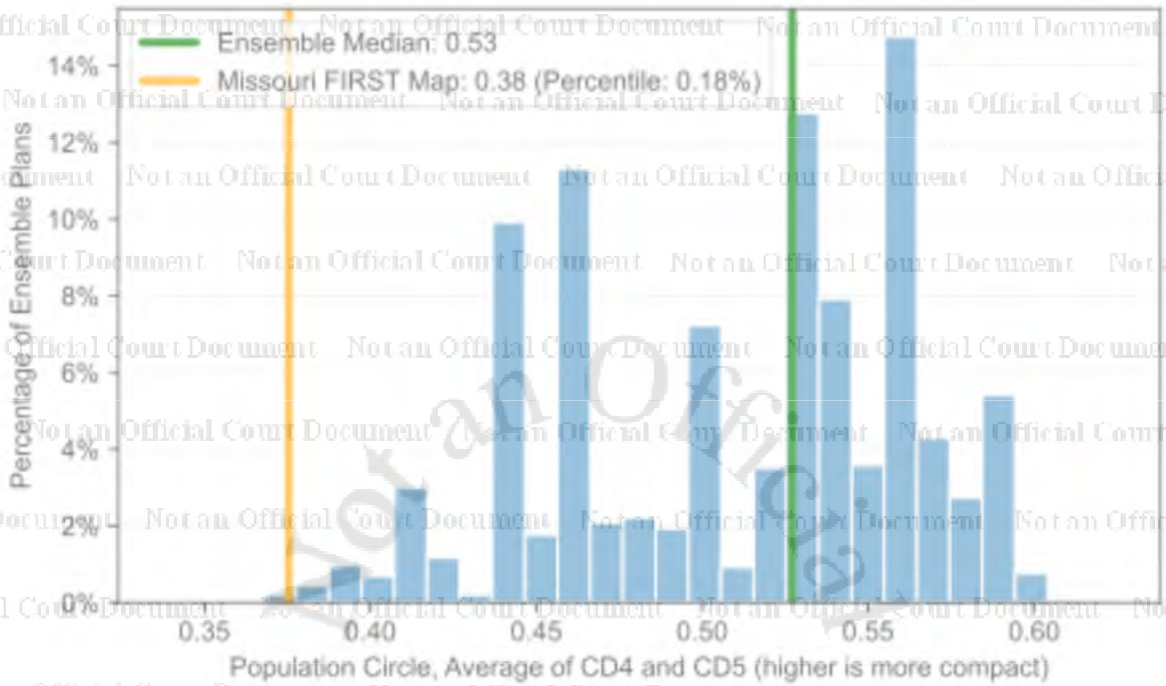


Figure 25: Histogram showing the average Ehrenburg score between CD4 and CD5.

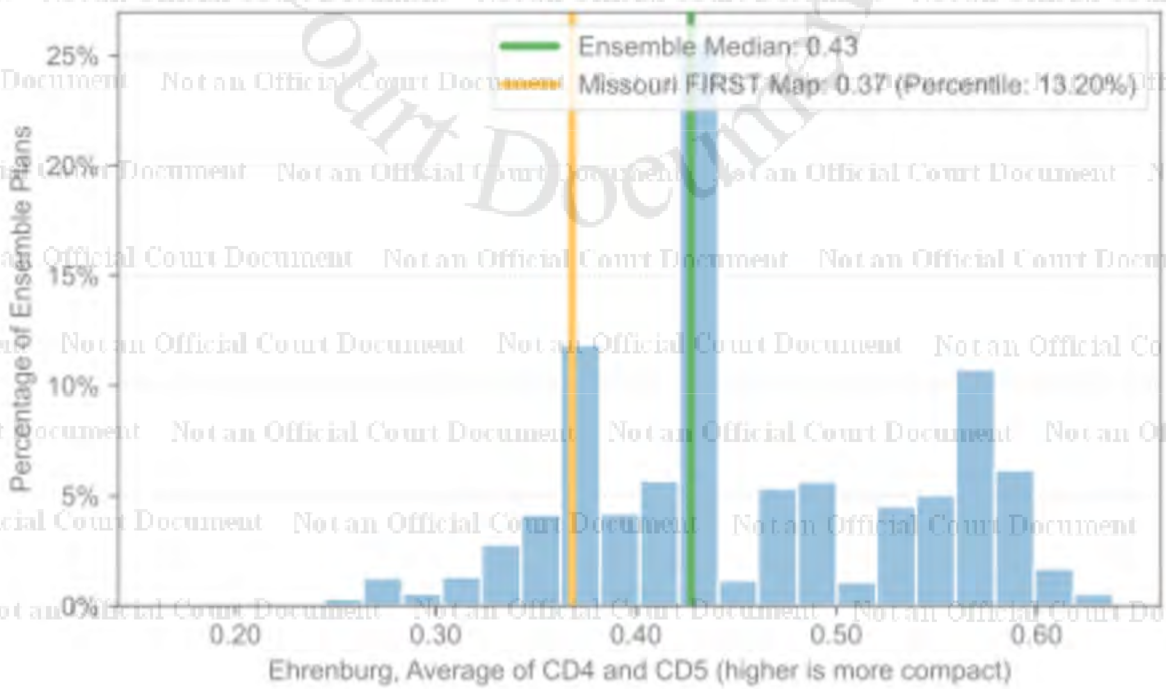


Figure 26: Histogram showing the average (Alternate) Schwartzberg score between CD4 and CD5.

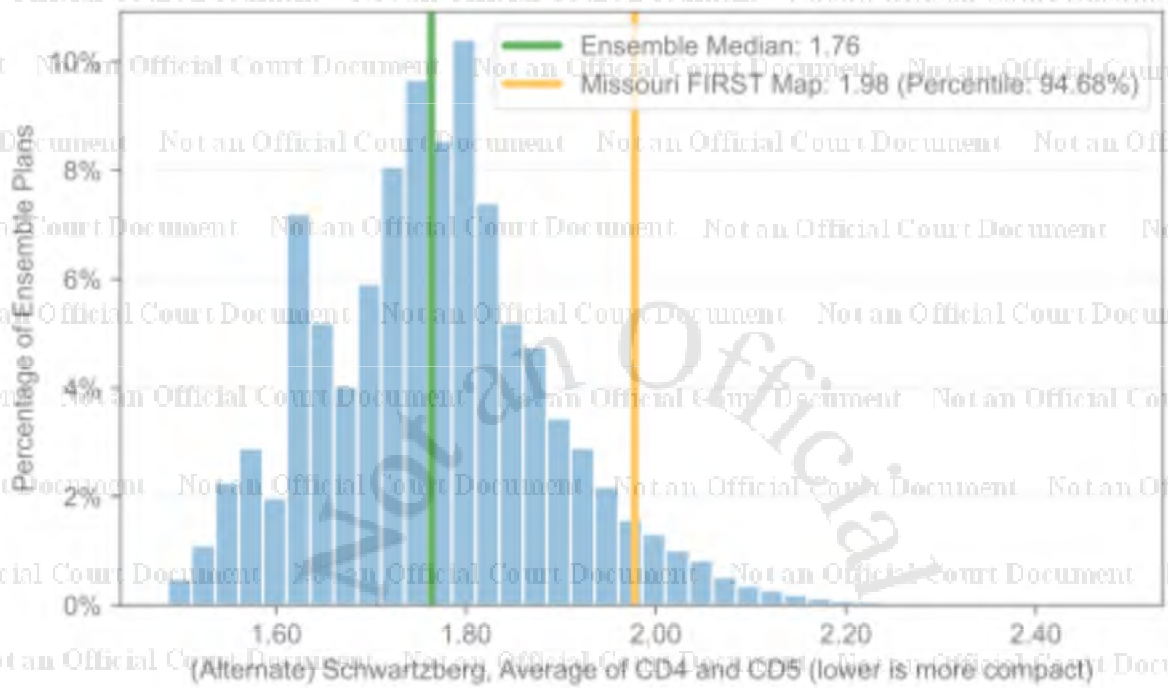


Figure 27: Histogram showing the average perimeter length between CD4 and CD5.

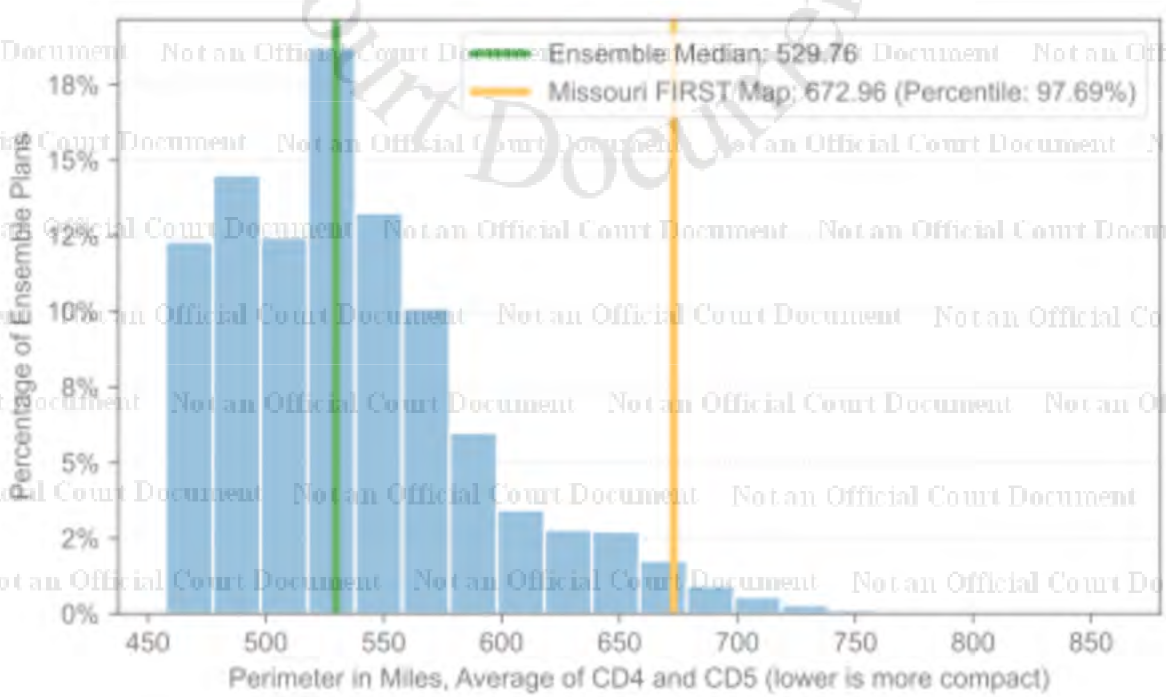
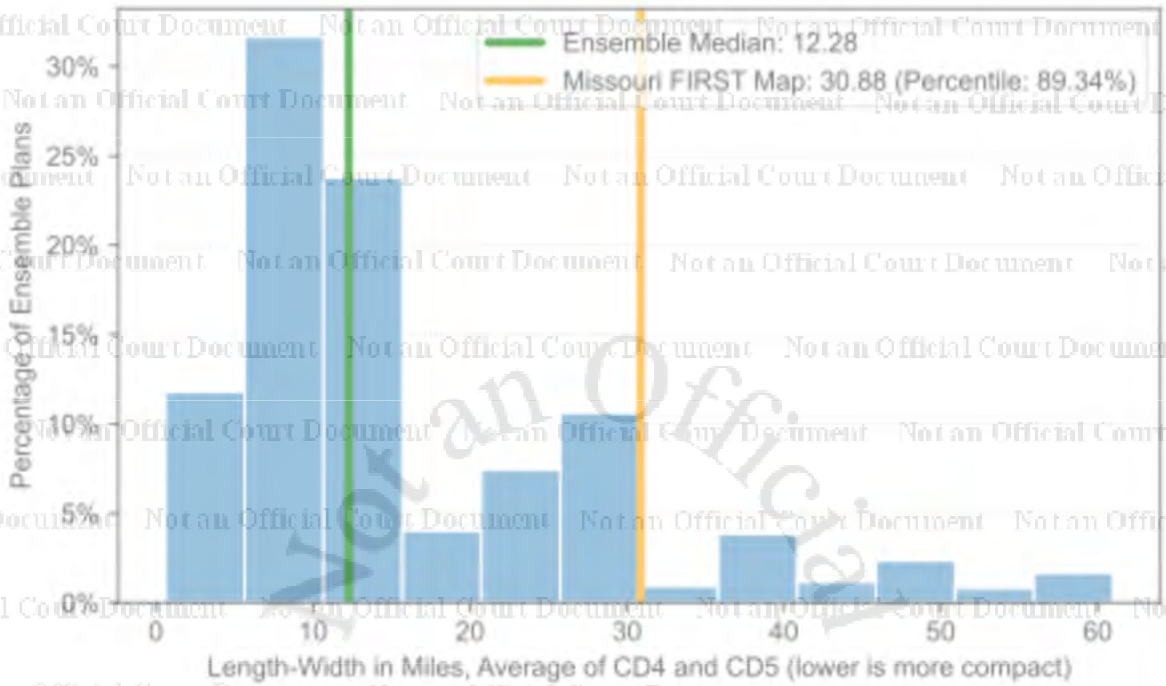


Figure 28: Histogram showing the average Length-Width score between CD4 and CD5.



APPENDIX 2: RESULTS FOR INCUMBENCY-CONSTRAINED ENSEMBLE MAPS

Table 10: County splitting between CD4 and CD5.

	Ensemble Median	Missouri FIRST Map	Percentile
Counties Split	1	1	51.04%
Population of Largest Jackson County Piece	701,167	370,868	0.11%
Total Population of Largest County Pieces	1,506,196	1,208,429	0.11%

Table 11: Municipality splitting between CD4 and CD5.

	Ensemble Median	Missouri FIRST Map	Percentile
Municipalities Split	6	6	55.24%
Population of Largest Kansas City Piece	300,419	174,515	0.28%
Total Population of Largest Municipality Pieces	1,075,487	960,864	0.97%

Table 12: VTD splitting between CD4 and CD5.

	Ensemble Median	Missouri First Map	Percentile
VTDs Split	0	18	100.00%
Total Population of Largest VTD Pieces	1,538,728	1,534,476	0.00%

Table 13: Size of the CD4–CD5 boundary, as measured by length in miles and by cut edges.

	Ensemble Median	Missouri FIRST Map	Percentile
Length in Miles	140.14	273.37	98.35%
Cut Edges	353	792	98.58%

Table 14: Compactness metrics for which higher scores indicate greater compactness..

		Ensemble Median	Missouri FIRST Map	Percentile
Reock	min	0.34	0.35	52.93%
	avg	0.44	0.39	3.67%
	max	0.54	0.43	0.19%
Polsby–Popper	min	0.28	0.20	9.95%
	avg	0.32	0.27	7.64%
	max	0.36	0.33	22.13%
Population Polygon	min	0.62	0.60	18.36%
	avg	0.76	0.62	0.22%
	max	0.90	0.63	0.13%
Area/Convex Hull	min	0.74	0.70	29.90%
	avg	0.77	0.76	31.53%
	max	0.82	0.82	49.63%
Population Circle	min	0.29	0.35	91.24%
	avg	0.46	0.38	0.09%
	max	0.63	0.40	0.39%
Ehrenburg	min	0.32	0.25	29.82%
	avg	0.42	0.37	26.80%
	max	0.51	0.49	41.22%

Table 15: Compactness metrics for which lower scores indicate greater compactness.

		Ensemble Median	Missouri FIRST Map	Percentile
(Alternate) Schwartzberg	min	1.67	1.74	77.87%
	avg	1.80	1.98	92.67%
	max	1.90	2.22	90.05%
Perimeter in Miles	min	270.85	628.75	99.31%
	avg	539.73	672.96	98.35%
	max	795.53	717.16	9.38%
Length-Width in Miles	min	0.33	11.80	87.89%
	avg	12.51	30.88	85.77%
	max	24.68	49.95	75.71%

Table 16: Splitting of 2012 and 2022 Congressional districts between CD4 and CD5.

	Ensemble Median	Missouri First Map	Percentile
2012 Congressional Districts Split	2	4	99.68%
Total Population of Largest District Pieces	1,450,191	1,007,069	0.35%
2022 Congressional Districts Split	2	3	80.34%
Total Population of Largest District Pieces	1,375,064	1,006,066	0.54%

Table 17: Splitting of Missouri Senate districts between CD4 and CD5.

	Ensemble Median	Missouri First Map	Percentile
State Senate Districts Split	2	5	92.80%
Total Population of Largest State Senate District Pieces	1,468,500	1,382,297	9.09%

Table 18: BVAP in the lower-BVAP district (CD4 in the Missouri FIRST Map) and higher-BVAP district (CD5 in the Missouri FIRST Map).

	Ensemble Median	Missouri FIRST Map	Percentile
BVAP: Lower of CD4 and CD5	25,708	45,836	95.65%
BVAP: Higher of CD4 and CD5	125,904	105,776	4.36%

APPENDIX 3: RESULTS FOR A STRICTER-POPULATION-BALANCE ENSEMBLE

Table 19: County splitting between CD4 and CD5.

	Ensemble Median	Missouri FIRST Map	Percentile
Counties Split	1	1	50.00%
Population of Largest Jackson County Piece	701,167	370,868	0.20%
Total Population of Largest County Pieces	1,498,890	1,208,429	0.20%

Table 20: Municipality splitting between CD4 and CD5.

	Ensemble Median	Missouri FIRST Map	Percentile
Municipalities Split	4	6	78.46%
Population of Largest Kansas City Piece	300,523	174,515	0.46%
Total Population of Largest Municipality Pieces	1,075,501	960,864	1.25%

Table 21: VTD splitting between CD4 and CD5.

	Ensemble Median	Missouri First Map	Percentile
VTDs Split	0	18	100.00%
Total Population of Largest VTD Pieces	1,538,728	1,534,476	0.00%

Table 22: Size of the CD4–CD5 boundary, as measured by length in miles and by cut edges.

	Ensemble Median	Missouri FIRST Map	Percentile
Length in Miles	131.60	273.37	97.46%
Cut Edges	340.00	792.00	97.35%

Table 23: Compactness metrics for which higher scores indicate greater compactness.

		Ensemble Median	Missouri FIRST Map	Percentile
Reock	min	0.40	0.35	23.57%
	avg	0.48	0.39	1.18%
	max	0.56	0.43	0.58%
Polsby–Popper	min	0.30	0.20	6.13%
	avg	0.33	0.27	6.47%
	max	0.36	0.33	25.25%
Population Polygon	min	0.61	0.60	23.95%
	avg	0.77	0.62	0.29%
	max	0.92	0.63	0.17%
Area/Convex Hull	min	0.79	0.70	17.41%
	avg	0.83	0.76	20.22%
	max	0.85	0.82	23.76%
Population Circle	min	0.29	0.35	91.29%
	avg	0.53	0.38	0.20%
	max	0.77	0.40	0.52%
Ehrenburg	min	0.40	0.25	13.63%
	avg	0.43	0.37	13.90%
	max	0.52	0.49	21.59%

Table 24: Compactness metrics for which lower scores indicate greater compactness.

		Ensemble Median	Missouri FIRST Map	Percentile
(Alternate) Schwartzberg	min	1.67	1.74	74.75%
	avg	1.76	1.98	94.33%
	max	1.83	2.22	93.87%
Perimeter in Miles	min	237.84	628.75	99.27%
	avg	531.19	672.96	97.46%
	max	805.61	717.16	4.15%
Length-Width in Miles	min	0.33	11.80	91.77%
	avg	12.00	30.88	88.42%
	max	23.23	49.95	79.69%

Table 25: Splitting of 2012 and 2022 Congressional districts between CD4 and CD5.

	Ensemble Median	Missouri First Map	Percentile
2012 Congressional Districts Split	2	4	99.72%
Total Population of Largest District Pieces	1,414,145	1,007,069	1.42%
2022 Congressional Districts Split	2	3	81.38%
Total Population of Largest District Pieces	1,379,157	1,006,066	1.39%

Table 26: Splitting of Missouri Senate districts between CD4 and CD5.

	Ensemble Median	Missouri First Map	Percentile
State Senate Districts Split	2	5	92.34%
Total Population of Largest State Senate District Pieces	1,470,037	1,382,297	12.03%

Table 27: BVAP in the lower-BVAP district (CD4 in the Missouri FIRST Map) and higher-BVAP district (CD5 in the Missouri FIRST Map).

	Ensemble Median	Missouri FIRST Map	Percentile
BVAP: Lower of CD4 and CD5	25,376	45,836	96.83%
BVAP: Higher of CD4 and CD5	126,236	105,776	3.17%

APPENDIX 4: ALTERNATIVE MAPS OBSERVED IN THE ENSEMBLES

Among the 100,000 maps in the ensemble with stronger population balance (±0.1%) are 30 maps that are, by chance, *exactly* population-balanced between CD4 and CD5. Of these, 11 maps also satisfy the incumbency condition, drawing the current CD4 and CD5 incumbents into separate districts. (Other maps in the ensemble that have some population deviation can be easily adjusted to achieve perfectly equal population without significantly affecting compactness or other relevant properties, as demonstrated by the adjustments made to several of my ensemble maps in a report produced by a different Plaintiffs' expert.)

This appendix presents the five most compact of these 11 maps as measured by cut edges; the maps are numbered as they occur in the ensemble and listed in order. The first of these, Map #12064, also happens to appear in the original ensemble with ±1% population tolerance. Since the maps are taken from the stricter-population-balance ensemble, the medians and percentiles stated in this appendix will also be relative to that ensemble, as presented in Appendix 3, rather than the original ensemble presented in the body of the report.

Some notes on the properties of these five maps:

a. Compared with the Missouri FIRST Map, these maps (like all the ensemble maps) vary only the CD4–CD5 boundary and leave the other six districts unchanged.

b. Four of the maps (all but #85776) split only Jackson County between CD4 and CD5, introducing no new split counties and thus keeping the three-way split of Jackson County between CD4, CD5, and CD6. Map #85776 splits a small portion of Cass County. All five maps are far superior to the Missouri FIRST Map in preservation of intact county population.

c. Four of the maps (all but #88159) split 7 municipalities between CD4 and CD5, more than the 6 splits in the Missouri FIRST Map. However, these splits are far less severe and preserve much more of the population of municipalities than the Missouri FIRST Map. For instance, the four maps that split Kansas City only split off between 170 and 6,424 Kansas City residents, compared to over 126,000 in the Missouri FIRST Map. In fact, the four maps splitting 7 municipalities preserve more of the total municipal population than Map #88159, which only has 3 municipality splits.

d. Four of the maps (all but #85776) split *zero* VTDs. Map #85776 splits two VTDs and about 900 people, whereas the Missouri FIRST Map splits 18 VTDs and over 4,000 people.

e. In all five maps, the length of the CD4–CD5 boundary is about half the size as in the Missouri FIRST Map, whether measured by miles or by cut edges.

f. Across the 9 district-by-district compactness metrics, the five maps generally compare favorably, without exhibiting the consistent and occasionally extreme non-compactness of the Missouri FIRST Map. Three of the maps (#12064, #26709, and #58210) have *all* of the compactness scores between the 10th and 90th percentiles, and a fourth (#85776) has only one score outside this range (minimum Population Polygon, 92.37%). Of the five, only Map #88159 has any significant compactness outliers compared to the ensemble.

g. All five maps far better preserve the historical Congressional district lines and current Missouri Senate district lines than does the Missouri FIRST Map.

h. BVAP is much less cracked in these five ensemble maps, compared with the Missouri FIRST Map. Of the five, Map #26709 has the lowest CD5 BVAP with 119,416—compared with only 105,776 in the Missouri FIRST Map CD5.

These maps further illustrate that the extreme non-compactness of the Missouri FIRST Map is not necessary to achieve perfect population balance, to separate incumbents, to achieve BVAP representation, to prevent splitting of political or historical boundaries, or to meet any of the other criteria on which these maps outperform the Missouri FIRST Map.

Figure 29: Map #12064

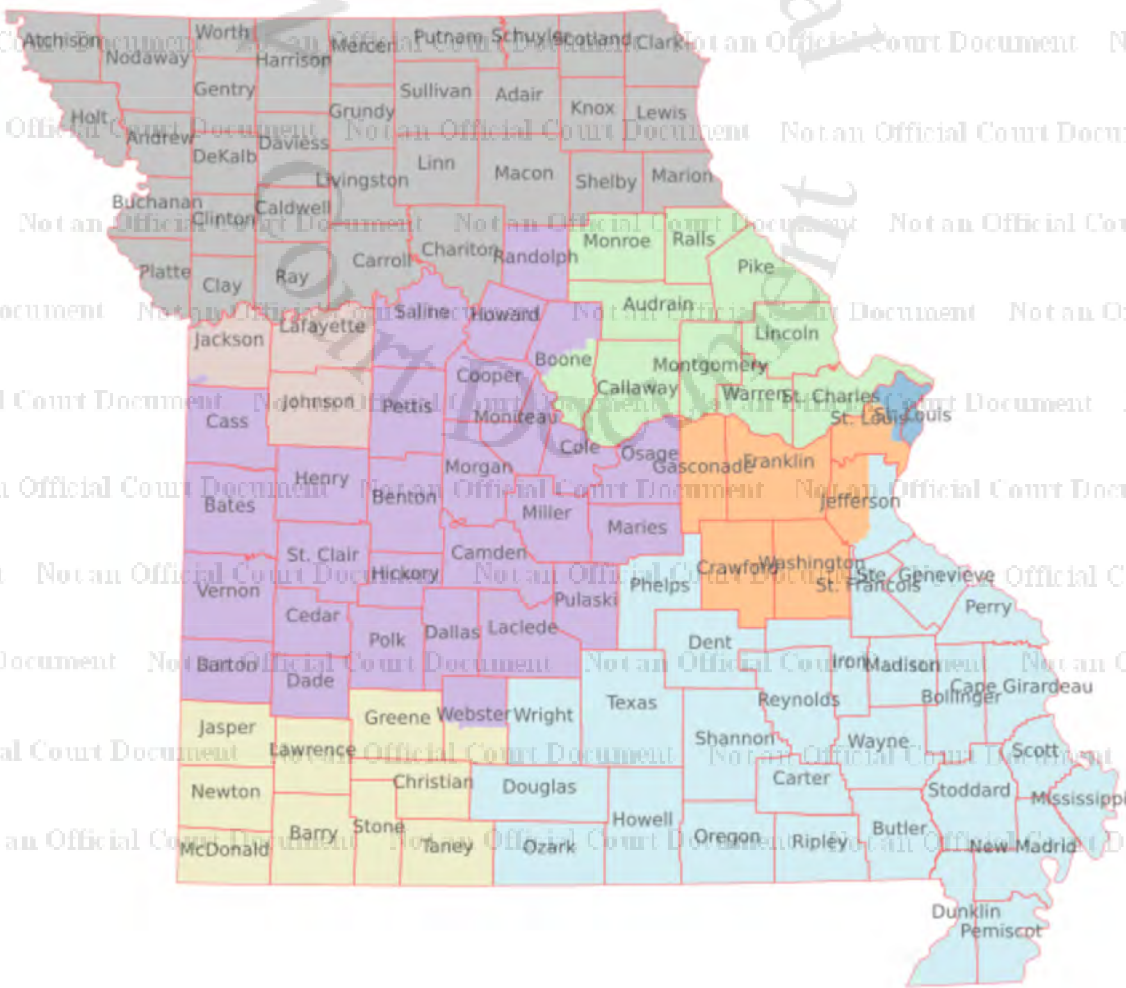


Figure 30: Map #26709

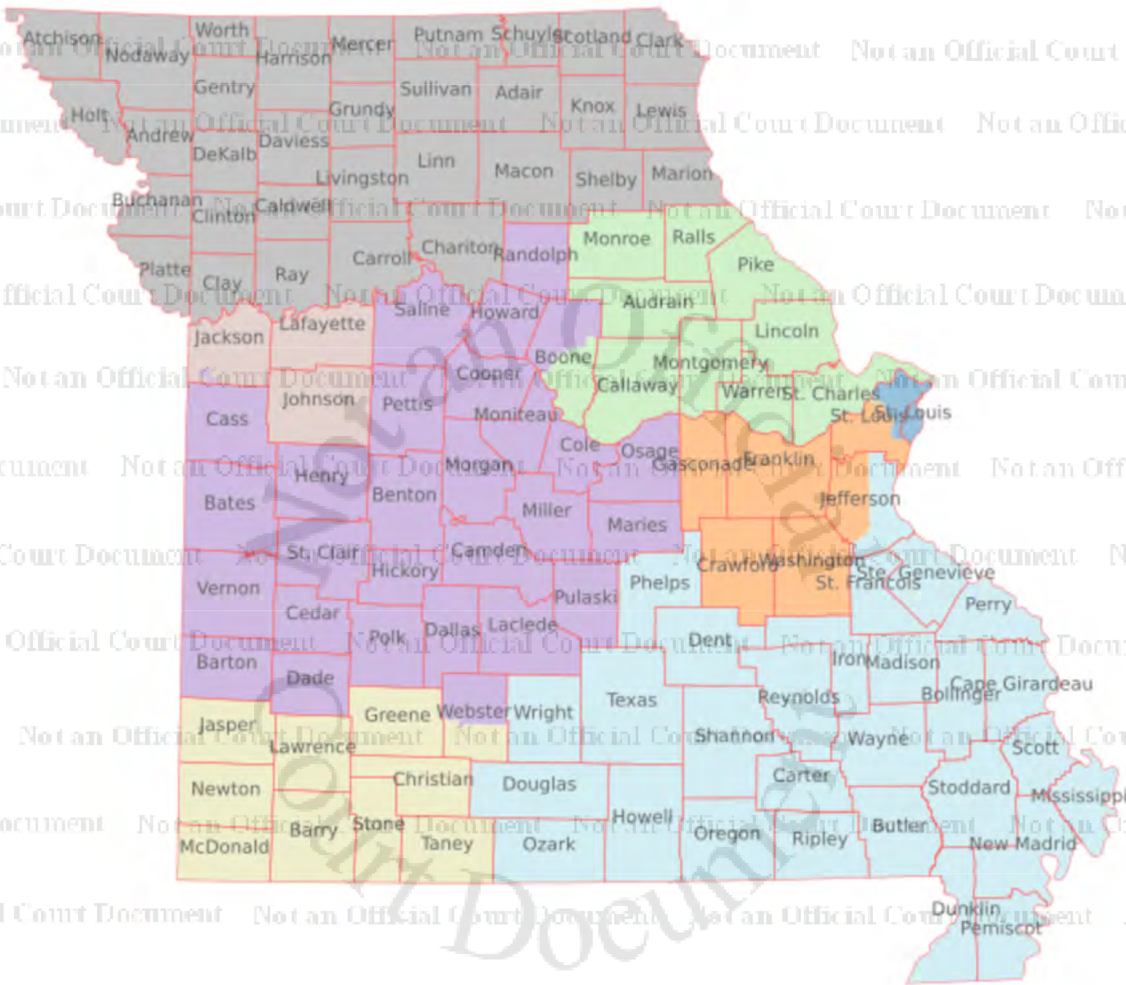


Figure 31: Map #58210

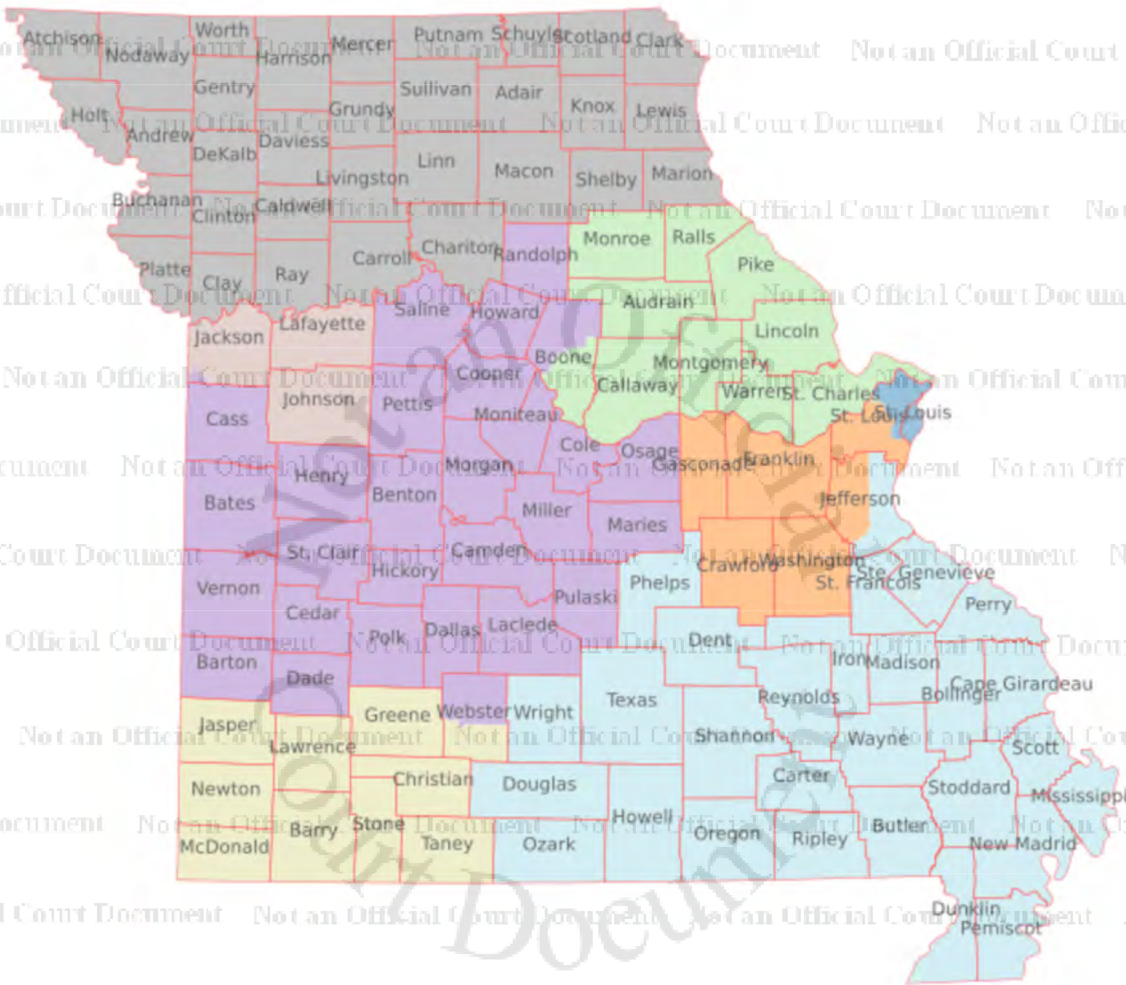


Figure 32: Map #85776

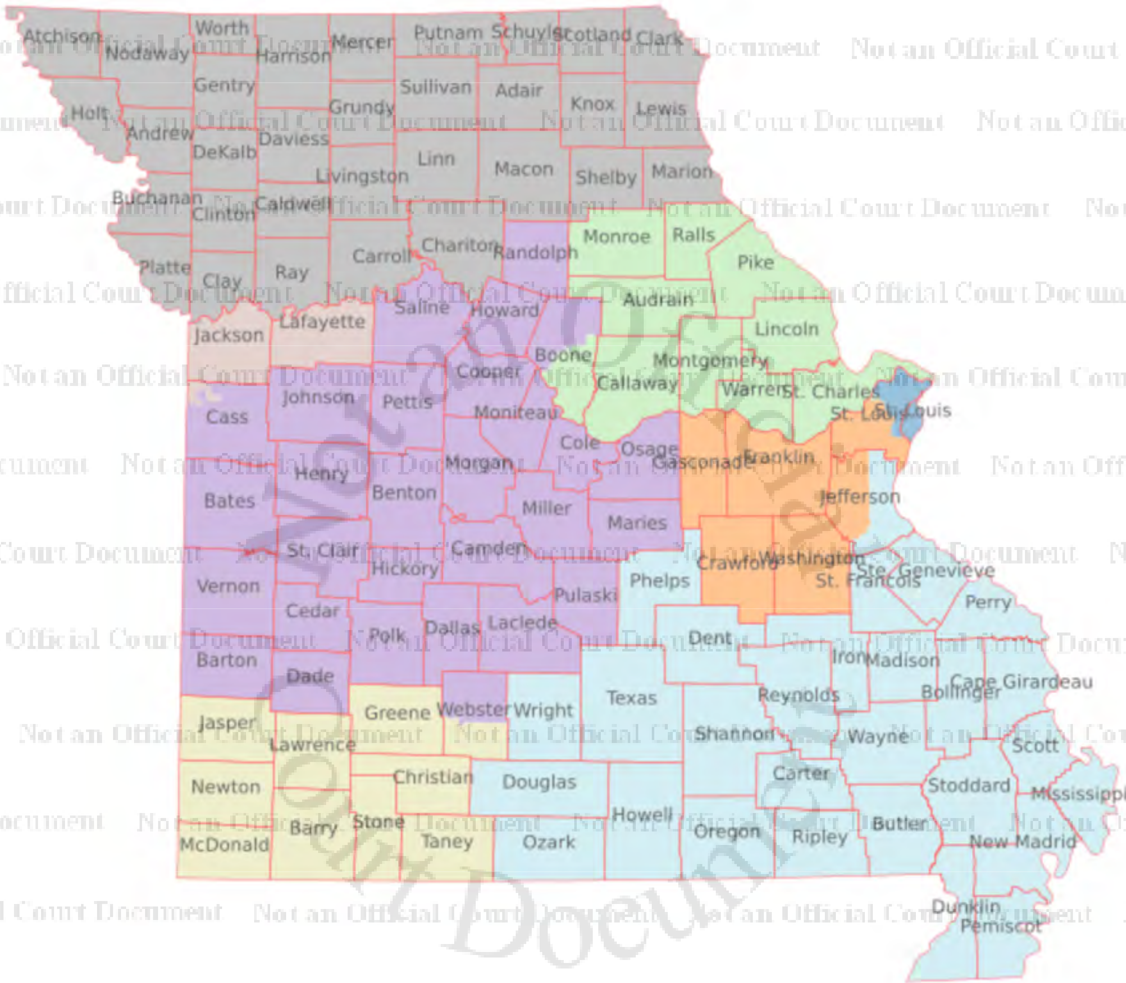
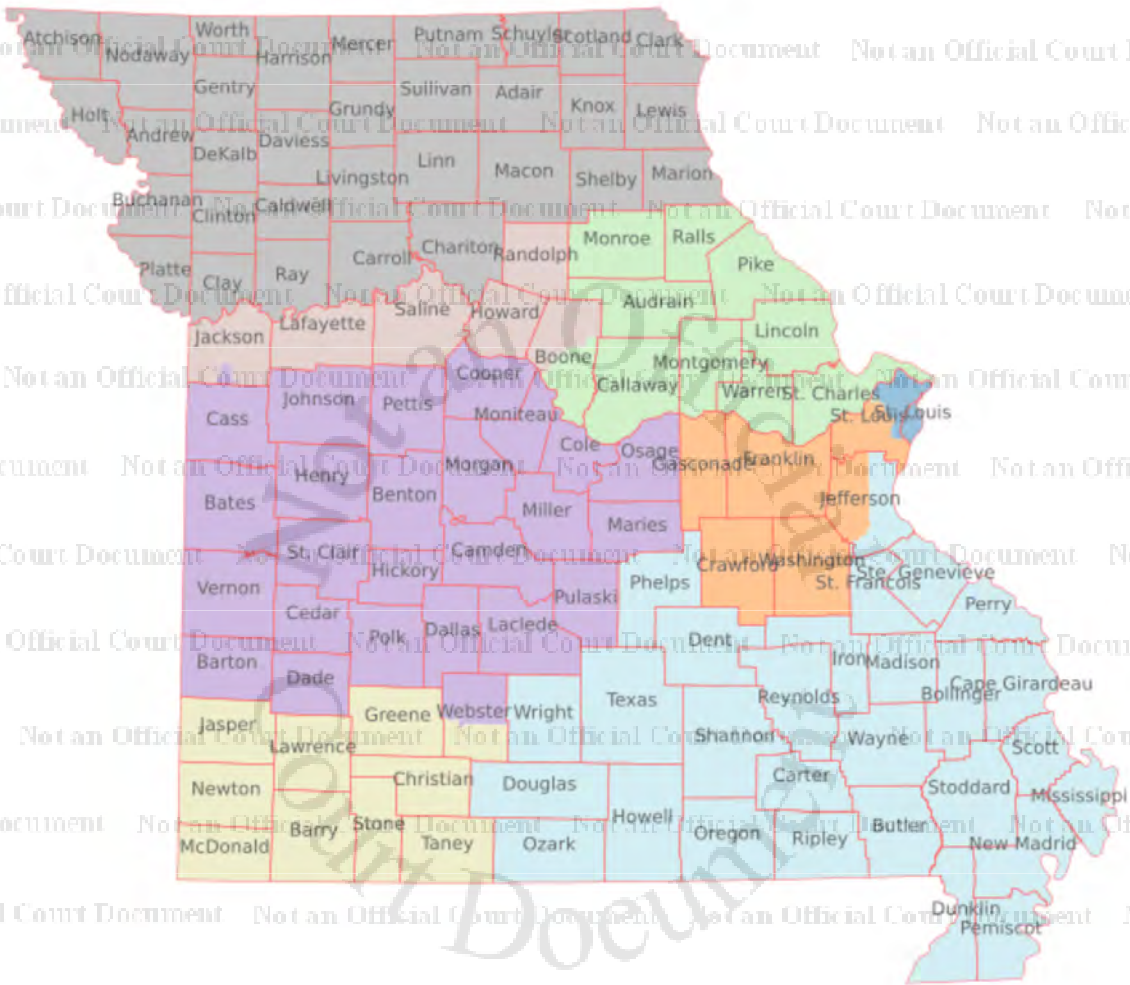


Figure 33: Map #88159



		Ensemble Median	Missouri FIRST Map	12064	26709	58210	85776	88159
Counties Split	Value	1	1	1	1	1	1	1
	Percentile	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%
Population of Largest Jackson County Piece	Value	701,167	370,868	682,367	682,367	682,367	701,167	645,150
	Percentile	73.56%	0.20%	43.99%	43.99%	43.99%	73.56%	20.61%
Total Population of Largest County Pieces	Value	1,498,890	1,208,429	1,519,928	1,519,928	1,519,928	1,503,515	1,482,711
	Percentile	49.90%	0.20%	90.07%	90.07%	90.07%	77.35%	20.61%
Municipalities Split	Value	4	6	7	7	7	7	3
	Percentile	45.89%	78.46%	90.71%	90.71%	90.71%	90.71%	27.08%
Population of Largest Kansas City Piece	Value	300,523	174,515	299,153	294,099	295,601	300,523	300,353
	Percentile	66.60%	0.46%	13.77%	9.46%	10.18%	66.60%	16.38%
Total Population of Largest Municipality Pieces	Value	1,075,501	960,864	1,074,346	1,065,487	1,067,242	1,069,244	1,039,033
	Percentile	50.00%	1.25%	47.12%	31.34%	34.59%	37.28%	11.68%
VTDs Split	Value	0	18	0	0	0	2	0
	Percentile	30.49%	100.00%	30.49%	30.49%	30.49%	84.82%	30.49%
Total Population of Largest VTD Pieces	Value	1,538,728	1,534,476	1,538,728	1,538,728	1,538,728	1,537,832	1,538,728
	Percentile	69.51%	0.00%	69.51%	69.51%	69.51%	1.70%	69.51%

		Ensemble Median	Missouri FIRST Map	12064	26709	58210	85776	88159
Length in Miles	Value	131.60	273.37	140.88	151.35	141.83	124.85	159.8
	Percentile	50.00%	97.46%	57.58%	65.02%	58.70%	43.96%	69.55%
Cut Edges	Value	340	792	369	432	366	356	425
	Percentile	49.86%	97.35%	59.35%	74.25%	58.48%	55.11%	73.22%

Minimum Compactness Score (higher is more compact)		Ensemble Median	Missouri FIRST Map	12064	26709	58210	**85776**	**88159**
Reock	Value	0.40	0.35	0.50	0.50	0.50	0.32	0.22
	Percentile	50.05%	23.57%	74.53%	75.25%	72.46%	15.72%	1.26%
Polsby–Popper	Value	0.30	0.20	0.31	0.30	0.31	0.26	0.17
	Percentile	50.00%	6.13%	62.42%	55.77%	61.60%	21.77%	1.56%
Population Polygon	Value	0.61	0.60	0.58	0.58	0.56	0.67	0.78
	Percentile	50.03%	23.95%	18.42%	20.05%	10.72%	92.37%	99.88%
Area/Convex Hull	Value	0.79	0.70	0.79	0.79	0.79	0.75	0.66
	Percentile	50.00%	17.41%	51.92%	52.91%	50.35%	30.24%	7.47%
Population Circle	Value	0.29	0.35	0.29	0.29	0.29	0.29	0.39
	Percentile	49.99%	91.29%	38.08%	38.08%	38.08%	38.08%	95.85%
Ehrenburg	Value	0.40	0.25	0.40	0.40	0.40	0.32	0.17
	Percentile	50.00%	13.63%	54.67%	55.33%	52.68%	27.18%	1.13%

Average Compactness Score (higher is more compact)		Ensemble Median	Missouri FIRST Map	12064	26709	58210	**85776**	**88159**
Reock	Value	0.48	0.39	0.52	0.52	0.52	0.44	0.40
	Percentile	50.02%	1.18%	68.90%	69.75%	66.68%	20.11%	2.33%
Polsby–Popper	Value	0.33	0.27	0.35	0.33	0.35	0.30	0.33
	Percentile	50.07%	6.47%	71.18%	54.49%	70.40%	24.22%	57.04%
Population Polygon	Value	0.77	0.62	0.74	0.73	0.73	0.80	0.80
	Percentile	49.92%	0.29%	23.20%	19.75%	18.84%	88.99%	89.10%
Area/Convex Hull	Value	0.83	0.76	0.81	0.80	0.80	0.79	0.77
	Percentile	50.00%	20.22%	45.02%	40.41%	43.68%	34.00%	26.51%
Population Circle	Value	0.53	0.38	0.50	0.50	0.50	0.46	0.42
	Percentile	50.00%	0.20%	35.33%	35.33%	35.33%	18.33%	6.84%
Ehrenburg	Value	0.43	0.37	0.43	0.42	0.43	0.42	0.36
	Percentile	50.00%	13.90%	39.52%	37.39%	41.86%	32.84%	12.19%

Maximum Compactness Score (higher is more compact)		Ensemble Median	Missouri FIRST Map	12064	26709	58210	85776	88159
Reock	Value	0.56	0.43	0.54	0.54	0.54	0.56	0.57
	Percentile	49.99%	0.58%	20.65%	19.89%	23.16%	47.37%	60.46%
Polsby–Popper	Value	0.36	0.33	0.39	0.35	0.39	0.34	0.49
	Percentile	49.96%	25.25%	67.80%	47.59%	66.34%	35.87%	91.11%
Population Polygon	Value	0.92	0.63	0.90	0.88	0.90	0.92	0.81
	Percentile	50.00%	0.17%	30.72%	21.77%	31.77%	47.69%	9.15%
Area/Convex Hull	Value	0.85	0.82	0.82	0.81	0.82	0.84	0.88
	Percentile	49.98%	23.76%	27.80%	22.95%	26.37%	40.72%	75.31%
Population Circle	Value	0.77	0.40	0.71	0.71	0.71	0.63	0.45
	Percentile	50.07%	0.52%	39.19%	39.19%	39.19%	18.96%	2.02%
Ehrenburg	Value	0.52	0.49	0.45	0.45	0.45	0.52	0.56
	Percentile	49.84%	21.59%	15.17%	14.24%	16.57%	51.07%	63.88%

Minimum Compactness Score (lower is more compact)		Ensemble Median	Missouri FIRST Map	12064	26709	58210	85776	88159
(Alternate) Schwartzberg	Value	1.67	1.74	1.60	1.68	1.60	1.70	1.43
	Percentile	50.04%	74.75%	32.20%	52.42%	33.66%	64.13%	8.89%
Perimeter in Miles	Value	237.84	628.75	257.17	270.65	258.11	251.66	488.32
	Percentile	50.00%	99.27%	58.21%	64.26%	58.81%	55.31%	93.77%
Length-Width in Miles	Value	0.33	11.80	0.33	0.33	0.33	0.33	37.33
	Percentile	43.75%	91.77%	43.75%	43.75%	43.75%	43.75%	97.47%

Average Compactness Score (lower is more compact)		Ensemble Median	Missouri FIRST Map	12064	26709	58210	85776	88159
(Alternate) Schwartzberg	Value	1.76	1.98	1.70	1.75	1.70	1.84	1.91
	Percentile	50.00%	94.33%	28.04%	44.06%	29.79%	74.60%	87.85%
Perimeter in Miles	Value	531.19	672.96	540.48	550.94	541.42	524.45	559.39
	Percentile	50.00%	97.46%	57.58%	65.02%	58.70%	43.96%	69.55%
Length-Width in Miles	Value	12.00	30.88	5.32	5.35	5.32	11.62	59.40
	Percentile	50.02%	88.42%	13.38%	15.80%	13.38%	48.93%	99.67%

Maximum Compactness Score (lower is more compact)		Ensemble Median	Missouri FIRST Map	12064	26709	58210	85776	88159
(Alternate) Schwartzberg	Value	1.83	2.22	1.80	1.82	1.80	1.97	2.40
	Percentile	50.00%	93.87%	37.58%	44.23%	38.40%	78.23%	98.44%
Perimeter in Miles	Value	805.61	717.16	823.78	831.23	824.72	797.23	630.45
	Percentile	49.53%	4.15%	64.05%	69.61%	65.13%	35.79%	0.58%
Length-Width in Miles	Value	23.23	49.95	10.30	10.36	10.30	22.91	81.48
	Percentile	50.03%	79.69%	13.39%	15.81%	13.39%	49.14%	98.44%

		Ensemble Median	Missouri FIRST Map	12064	26709	58210	85776	88159
2012 Congressional Districts Split	Value	2	4	2	2	2	2	3
	Percentile	34.76%	99.72%	34.76%	34.76%	34.76%	34.76%	81.67%
Total Population of Largest District Pieces	Value	1,414,145	1,007,069	1,442,582	1,442,582	1,442,582	1,480,182	1,414,814
	Percentile	49.63%	1.42%	75.96%	75.96%	75.96%	85.32%	65.96%
2022 Congressional Districts Split	Value	2	3	3	3	3	2	3
	Percentile	31.57%	81.38%	81.38%	81.38%	81.38%	31.57%	81.38%
Total Population of Largest District Pieces	Value	1,379,157	1,006,066	1,341,965	1,341,965	1,341,965	1,379,565	1,305,550
	Percentile	49.95%	1.39%	27.94%	27.94%	27.94%	65.36%	17.27%
State Senate Districts Split	Value	2	5	3	4	3	2	3
	Percentile	45.80%	92.34%	68.72%	84.50%	68.72%	45.80%	68.72%
Total Population of Largest State Senate District Pieces	Value	1,470,037	1,382,297	1,432,931	1,432,931	1,432,931	1,470,531	1,465,608
	Percentile	49.86%	12.03%	40.39%	40.39%	40.39%	64.10%	45.59%

		Ensemble Median	Missouri FIRST Map	12064	26709	58210	85776	88159
BVAP: Lower of CD4 and CD5	Value	25,376	45,836	30,351	32,196	29,055	25,080	26,924
	Percentile	50.00%	96.83%	88.88%	90.80%	86.04%	42.96%	78.64%
BVAP: Higher of CD4 and CD5	Value	126,236	105,776	121,261	119,416	122,557	126,532	124,688
	Percentile	50.01%	3.17%	11.12%	9.20%	13.96%	57.04%	21.36%

APPENDIX 5: Curriculum Vitae of Dr. Ari J. Stern

Ari Stern

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Curriculum Vitae

prepared December 22, 2025

Academic Positions

2024– **Professor**, *Washington University in St. Louis*
Department of Mathematics

2018–2024 **Associate Professor**, *Washington University in St. Louis*
Department of Mathematics

2012–2018 **Assistant Professor**, *Washington University in St. Louis*
Department of Mathematics

2009–2012 **Postdoctoral Scholar and Lecturer**, *University of California, San Diego*
Department of Mathematics

Education

2009 **Ph.D.**, *California Institute of Technology*
Applied and Computational Mathematics
Thesis: *Geometric discretization of Lagrangian mechanics and field theories*
Advisors: Jerrold E. Marsden and Mathieu Desbrun

2002 **M.A.**, *Columbia University*
Mathematics of Finance

2001 **B.A.**, *Columbia University*
Mathematics

Publications and Preprints

[36] A. Stern and E. Zampa, *Multisymplecticity in finite element exterior calculus*, Found. Comput. Math. (2025). doi:10.1007/s10208-025-09720-y.

[35] A. Stern and M. Viviani, *Quadratic projectable Runge–Kutta methods* (2024), in review. arXiv:2411.12634 [math.NA].

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Grants

2025–2030 *Structure-Preserving Numerical Methods for Hamiltonian PDEs*
Simons Foundation, Grant No. SFI-MPS-TSM-00014348
Role: PI
Award Amount: $42,000

2022–2025 *Structure-Preserving Hybrid Finite Element Methods*
National Science Foundation, Grant No. DMS-2208551
Role: PI
Award Amount: $237,648

2019–2023 *Hybrid Finite Element Methods for Geometric PDEs*
National Science Foundation, Grant No. DMS-1913272
Role: PI
Award Amount: $212,640

2018–2023 *Imaging Tauopathy in the Dominantly Inherited Alzheimer Network (DIAN)*
National Institutes of Health, Grant No. 5R01AG05255003
Role: Personnel
Award Amount: $3,583,474

2013–2018 *Collaborative Research in Geometric Numerical Analysis*
Simons Foundation, Grant No. 279968
Role: PI
Award Amount: $35,000

2011–2013 AMS–Simons Travel Grant
American Mathematical Society and Simons Foundation
Role: PI
Award Amount: $4,000

Visiting Positions

Jul–Dec 2019 University of Cambridge, Isaac Newton Institute for Mathematical Sciences

Jun–Aug 2014 Massey University, Palmerston North, New Zealand

Sep–Dec 2009 CSIC (Spanish National Research Council), Madrid

Fellowships

Mar 2016 & Jul 2012 US Junior Oberwolfach Fellowship
Mathematisches Forschungsinstitut Oberwolfach, NSF Grant No. DMS-1049268
Role: Travel Support Recipient

2009–2011 Center for Theoretical Biological Physics Postdoctoral Fellowship
University of California, San Diego
Role: Postdoctoral Fellow

Sep–Dec 2009 SIMUMAT Visiting Research Fellowship
CSIC (Spanish National Research Council), Madrid, Spain
Role: Visiting Postdoctoral Fellow

2003–2007 Gordon and Betty Moore Foundation Fellowship
California Institute of Technology
Role: Graduate Fellow

2000–2001 VIGRE Undergraduate Research Fellowship
Columbia University, NSF Grant No. 9810750
Role: Undergraduate Fellow

Awards and Honors

2009 W. P. Carey & Co., Inc., Prize in Applied Mathematics
California Institute of Technology
"For an outstanding doctoral dissertation in applied mathematics or pure mathematics"

2008 Everhart Distinguished Graduate Lecture Award
California Institute of Technology

2001 John Dash Van Buren, Jr., Prize in Mathematics
Columbia University

2000 Professor Van Amringe Mathematical Prize
Columbia University

Professional Talks

Plenary Addresses

Dec 2, 2024 Lie–Størmer Colloquium, Tromsø
Jul 25, 2022 SciCADE Conference, Reykjavik
Jun 30, 2022 *Acta Numerica* 30th Anniversary Conference, Będlewo
Sep 10, 2021 NUMDIFF Conference, Halle
Mar 28, 2014 Mathematical Association of America (MAA) Missouri Section Meeting, St. Louis
Jan 12, 2010 Fourth International Young Researchers' Workshop on Geometry, Mechanics, and Control, Ghent

Invited Research Talks

Oct 19, 2025 AMS Fall Central Sectional Meeting, St. Louis
Jun 27, 2025 Delft University of Technology
Apr 29, 2025 Saint Louis University
Mar 19, 2025 Workshop on Geometric Mechanics Formulations for Continuum Mechanics, Banff International Research Station (BIRS)
Mar 7, 2025 SIAM Conference on Computational Science and Engineering (CSE25), Fort Worth
Oct 5, 2024 SIAM Central States Section Meeting, Kansas City
Jul 8, 2024 SIAM Annual Meeting, Spokane
Jun 13, 2024 University of Cambridge
Jun 10, 2024 Imperial College London
Jun 6, 2024 University of Oxford
Apr 14, 2024 Midwest Numerical Analysis Day, Iowa City

Jan 16, 2024	Workshop on Numerical Analysis and PDEs (WONAPDE), Concepción
Nov 17, 2023	Portland State University
Oct 24, 2023	Columbia University
Aug 24, 2023	International Congress on Industrial and Applied Mathematics (ICIAM), Tokyo
Jun 13, 2023	Foundations of Computational Mathematics (FoCM), Paris
Apr 15, 2023	AMS Spring Central Sectional Meeting, Cincinnati
Mar 1, 2023	SIAM Conference on Computational Science and Engineering (CSE23), Amsterdam
Oct 1, 2022	SIAM Central States Section Meeting, Stillwater
Jul 15, 2022	SIAM Annual Meeting, Pittsburgh
Jun 22, 2022	Workshop on Hilbert Complexes: Analysis, Applications, & Discretizations, Oberwolfach
Jun 13, 2022	Canadian Applied and Industrial Mathematics Society Annual Meeting
Jun 9, 2022	Seminario Internacional: Geometría Diferencial y Física Matemática, virtual
May 24, 2022	University of California, San Diego
May 5, 2022	University of Notre Dame
Apr 9, 2022	Finite Element Circus, University of Florida
Oct 2, 2021	SIAM Central States Section Meeting, virtual
Sep 18, 2021	SIAM Southeastern Atlantic Section Meeting, Auburn
Sep 15, 2021	Firedrake '21, virtual
Apr 1, 2021	Geometric Numerical Integration Workshop, Oberwolfach, virtual
Mar 19, 2021	University of Warwick, virtual
Mar 4, 2021	SIAM Conference on Computational Science and Engineering (CSE21), virtual
Nov 7, 2020	Finite Element Circus, virtual
Nov 3, 2020	Imperial College London, virtual
Oct 30, 2020	Brown University, virtual
Sep 30, 2020	University of Tennessee, virtual
Jul 7, 2020	International Conference on Mathematical Neuroscience (ICMNS), virtual
Jun 18, 2020	Foundations of Computational Mathematics (FoCM), virtual
Feb 17, 2020	Princeton University
Dec 12, 2019	Karlsruhe Institute of Technology
Nov 11, 2019	University of Oxford
Nov 8, 2019	Imperial College London
Oct 14, 2019	International Centre for Mathematical Sciences, Edinburgh
Sep 30, 2019	University of Cambridge
Aug 20, 2019	Imperial College London
Aug 14, 2019	University of Cambridge
Jul 22, 2019	SciCADE, Innsbruck
Apr 4, 2019	University of Iowa
Mar 23, 2019	Finite Element Circus, Purdue
Jan 25, 2019	Missouri University of Science and Technology
Nov 2, 2018	University of Missouri–St. Louis
Oct 21, 2018	AMS Fall Central Sectional Meeting, Ann Arbor

Oct 6, 2018	SIAM TX-LA Sectional Meeting, Baton Rouge
July 25, 2018	World Congress on Computational Mechanics, New York
Apr 5, 2018	Symmetry and Computations Workshop, CIRM, Marseille
Dec 11, 2017	University of Illinois, Urbana–Champaign
Sep 30, 2017	SIAM Central States Section Meeting, Fort Collins
Jul 11, 2017	Foundations of Computational Mathematics (FoCM), Barcelona
Jun 12, 2017	Workshop on Connections in Geometric Numerical Integration and Structure-Preserving Discretization, Banff International Research Station (BIRS)
Dec 4, 2016	Canadian Mathematical Society Winter Meeting, Niagara Falls
Nov 21, 2016	California Institute of Technology
Nov 16, 2016	University of California, Riverside
Oct 2, 2016	SIAM Central States Section Meeting, Little Rock
Aug 23, 2016	University of Bergen, Norway
Apr 11, 2015	SIAM Central States Section Meeting, Rolla
Mar 9, 2015	University of Illinois, Urbana–Champaign
Dec 17, 2014	Foundations of Computational Mathematics (FoCM), Montevideo
Dec 9, 2014	Penn State
Oct 30, 2014	Wesleyan University
Jul 10, 2014	Massey University, Palmerston North, New Zealand
May 19, 2014	California Institute of Technology
Jan 9, 2014	Australian National University
Oct 28, 2013	IMA Workshop on Modern Applications of Homology and Cohomology, Minneapolis
Oct 19, 2013	AMS Fall Central Sectional Meeting, St. Louis
Oct 17, 2013	University of Chicago
Mar 11, 2013	CRM–McGill Applied Mathematics Seminar, Montreal
Jan 11, 2013	Joint Mathematics Meetings, San Diego
Jul 27, 2012	Conference on Geometry, Symmetry, Dynamics, and Control: the Legacy of Jerry Marsden, Fields Institute, Toronto
Jul 12, 2012	Discrete Differential Geometry Workshop, Oberwolfach
Jan 27, 2012	University of Wisconsin, Madison
Dec 14, 2011	Washington University in St. Louis
Nov 14, 2011	SIAM Conference on Analysis of Partial Differential Equations (PD11), San Diego
Oct 29, 2011	"Gone Fishing" Poisson Geometry Meeting, St. Louis
Sep 2, 2011	University of California, Berkeley
Jul 27, 2011	US National Congress on Computational Mechanics (USNCCM), Minneapolis
Jul 21, 2011	International Congress on Industrial and Applied Mathematics (ICIAM), Vancouver
Jul 4, 2011	Foundations of Computational Mathematics (FoCM), Budapest
May 6, 2011	Penn State
Mar 24, 2011	Geometric Numerical Integration Workshop, Oberwolfach
Feb 18, 2011	University of Wisconsin, Madison
Jan 25, 2011	University of Maryland, College Park

Jun 1, 2010 Dynamical Systems and Partial Differential Equations (DSPDEs'10), Barcelona
Apr 27, 2010 Sixth Structured Integrators Workshop, San Diego
Apr 8, 2010 Massey University, Palmerston North, New Zealand
Nov 20, 2009 Universidad Complutense de Madrid
Nov 6, 2009 Universidad de La Laguna, Canary Islands, Spain
Sep 17, 2009 Real Academia de Ciencias (Royal Academy of Sciences), Madrid
Aug 28, 2009 University of Notre Dame
May 7, 2009 Fifth Structured Integrators Workshop, California Institute of Technology
Apr 9, 2009 University of California, San Diego
Nov 21, 2008 University of Cambridge
Jul 3, 2008 Progress in Electromagnetics Research Symposium (PIERS), Cambridge
Jun 17, 2008 University of California, San Diego
Apr 2, 2008 Everhart Lecture Series, California Institute of Technology
Mar 22, 2008 Pacific Coast Gravity Meeting (PCGM), University of California, Santa Barbara
Oct 11, 2007 University of Southern California
Aug 13, 2007 Workshop on Geometric Mechanics, Banff International Research Station (BIRS)
Jun 14, 2007; Jun 21, 2007; Aug 2, 2007 Three-Part Lecture Series on Geometric Discretization of Classical Physics, TAPIR Numerical Relativity Seminar, California Institute of Technology.
Apr 30, 2007 Third Structured Integrators Workshop, University of Southern California
Nov 13, 2006 Workshop on Geometry and Computer Graphics, Columbia University
Jul 30, 2006 Discrete Differential Geometry: An Applied Introduction, SIGGRAPH, Los Angeles

Research Supervision

Postdoctoral Advisor

2020–2022 Shuhao Cao
Current Position: Tenure-Track Assistant Professor, University of Missouri–Kansas City
2016–2019 Yakov Berchenko-Kogan
Current Position: Tenure-Track Assistant Professor, Florida Institute of Technology

Ph.D. Advisor

2024– Marston Xue
2023– Calvin Reedy
2021–2025 Sanah Suri
Thesis: *Functional Equivariance and Backward Error Analysis*
Current Position: Postdoctoral Fellow, University of California, Davis
2018–2022 Mary Barker
Thesis: *A Nonconforming Finite Element Method for the 2D Vector Laplacian*
Current Position: Teaching Assistant Professor, University of Minnesota Duluth

Undergraduate Research Advisor

2021–2022 Aidan Kelley (jointly co-advised with Xiang Tang)
2020–2021 Jiawei Hu
2016–2017 Zhengdao Chen

2015–2016	Alexander Tettenhorst
2013–2014	Evan Miller

Ph.D. Thesis Committee Member

2025–	Swarup Dhar
2025	Boris Andrews (University of Oxford)
2025	Ty Easley
2024	Eric Pasewark
2023	Kiprian Berbatov (University of Manchester)
2023	Yanjie Zhong
2022	Bowei Zhao
2022	Joshua Covey
2020	Luis Garcia German
2016	Benjamin Passer
2015	Casey Boyett
2015	Matthew Wallace
2013	Kelly Bickel
2013	Timothy Chumley

Teaching Experience

Washington University in St. Louis

Spring 2025	MATH 450, *Numerical Methods for Differential Equations*
Fall 2024	MATH 449, *Numerical Applied Mathematics*
Spring 2024	MATH 5052, *Measure Theory & Functional Analysis II*
Fall 2023	MATH 5051, *Measure Theory & Functional Analysis I*
Spring 2023	MATH 4121, *Introduction to Lebesgue Integration*
Fall 2022	MATH 4111, *Introduction to Analysis*
Spring 2022	MATH 5052, *Measure Theory & Functional Analysis II*
Fall 2021	MATH 5051, *Measure Theory & Functional Analysis I*
Spring 2021	MATH 204, *Honors Mathematics II*
Fall 2020	MATH 203, *Honors Mathematics I*
	MATH 598, *Mathematical Professional Development*
Spring 2020	MATH 308, *Mathematics for the Physical Sciences*
	MATH 450, *Numerical Methods for Differential Equations*
Spring 2019	MATH 233, *Calculus III*
Fall 2018	MATH 547, *Geometric Mechanics*
Spring 2018	MATH 450, *Numerical Methods for Differential Equations*
Fall 2017	MATH 449, *Numerical Applied Mathematics*
	MATH 456, *Topics in Financial Mathematics*
Spring 2017	MATH 217, *Differential Equations*
Spring 2016	MATH 450, *Numerical Methods for Differential Equations*
Fall 2015	MATH 449, *Numerical Applied Mathematics*

	MATH 456, *Topics in Financial Mathematics*
Spring 2015	MATH 131, *Calculus I*
	MATH 450, *Numerical Methods for Differential Equations*
Fall 2014	MATH 449, *Numerical Applied Mathematics*
Spring 2014	MATH 515, *Partial Differential Equations*
Fall 2013	MATH 456, *Topics in Financial Mathematics*
Spring 2013	MATH 5052, *Measure Theory & Functional Analysis II*
Fall 2012	MATH 5051, *Measure Theory & Functional Analysis I*

University of California, San Diego

Spring 2012	MATH 10B, *Calculus II*
Winter 2012	MATH 142B, *Introduction to Analysis II*
Fall 2011	MATH 142A, *Introduction to Analysis I*
Spring 2011	MATH 20E, *Vector Calculus*
Winter 2011	MATH 20F, *Linear Algebra*
Fall 2010	MATH 10A, *Calculus I*

Academic Service

Departmental Service

2023–	Director of Undergraduate Studies
Spring 2022	Interim Director of Undergraduate Studies
2021– 2018–2019 2012–2015	Undergraduate Committee (Chair, Spring 2022 and 2023–)
2022– 2018–2019	Executive Committee
2020– 2016–2019 2012–2015	Computing Committee (Chair, 2014–2015 and 2020–2025)
2021–2022	Building Committee
2020–2021	Flexible Teaching and Seminars Committee (Chair)
2019–2021	Tenure-Track Search Committee (Chair, 2020–2021)
2019–2020 2014–2016	Postdoc Search Committee (Chair, 2015–2016 and 2019–2020)
2015–2019	Web Site Committee (Chair)
2013–2014	Math Club, Faculty Supervisor
2013–2015	Calculus Committee

School and University Service

2024–2025	"Literacies for Life and Career" Pilot Instructor
2023–2024	Academic Integrity Working Group on Grade Penalties
2017–2018	Advisory Group on IT Resources for Teaching and Learning

Other WashU Service

2022 Mathematical Contest in Modeling, Faculty Advisor

Workshops Organized

2027 *Structure Preserving Methods for Computational Geometric Mechanics*
Oberwolfach

2026 *Geometric Integration and Computational Mechanics*
FoCM'26, Vienna

2025 *Numerical PDEs and Geometry*
AMS Fall Central Sectional Meeting, St. Louis

2025 *DEC/FEEC: Discrete and Finite Element Exterior Calculus*
SIAM CSE25, Fort Worth

2023 *Exterior Calculus in Numerical Computing, Modeling, and Simulation*
SIAM CSE23, Amsterdam

2019 *Geometry and Structure Preservation in Numerical Differential Equations*
SciCADE, Innsbruck

Editorial Boards

2025– Associate Editor, *Foundations of Computational Mathematics*

2023– Associate Editor, *Geometric Mechanics*

2022– Associate Editor *International Journal of Numerical Analysis and Modeling*

2020–2023 Associate Editor, *Journal of Geometric Mechanics*

Peer Review

Acta Applicandae Mathematicae (Springer)
Advances in Computational Mathematics (Springer)
Applied Mathematics and Computation (Elsevier)
Applied Mathematics Letters (Elsevier)
Applied Numerical Mathematics (Elsevier)
BIT Numerical Mathematics (Springer)
Chinese Physics Letters (IOP)
Communications in Computational Physics (Global Science Press)
Communications in Mathematical Sciences (International Press)
Comptes Rendus Mecanique (Elsevier)
Computational and Applied Mathematics (Springer)
Computers and Mathematics with Applications (Elsevier)
Discrete and Continuous Dynamical Systems (AIMS)
Foundations of Computational Mathematics (Springer)
Games and Economic Behavior (Elsevier)
Geoscientific Model Development (Copernicus/EGU)
IMA Journal of Numerical Analysis (Oxford)
International Journal of Numerical Analysis and Modeling (Global Science Press)
Journal of Computational Dynamics (AIMS)
Journal of Computational Mathematics (Global Science Press)
Journal of Computational Physics (Elsevier)

Journal of the European Mathematical Society (EMS)
Journal of Geometric Mechanics (AIMS)
Journal of Nonlinear Science (Springer)
Journal of Physics A: Mathematical and Theoretical (IOP)
Mathematics of Computation (AMS)
Numerical Algorithms (Springer)
Numerische Mathematik (Springer)
Physics Letters A (Elsevier)
Proceedings of the London Mathematical Society (LMS/Wiley)
SIAM Journal on Applied Algebra and Geometry (SIAM)
SIAM Journal on Numerical Analysis (SIAM)
SIAM Journal on Scientific Computing (SIAM)
SMAI Journal of Computational Mathematics (SMAI)
Zeitschrift für Angewandte Mathematik und Physik (Birkhäuser)

Grant Review Panels

National Science Foundation
Simons Foundation

Professional Memberships

American Mathematical Society (AMS)
Society for Industrial and Applied Mathematics (SIAM)

Public Service Work on Mathematics of Redistricting

2017– Collaboration with MGGG Redistricting Lab (PI: Moon Duchin, Tufts)

2022 Public Testimony, Missouri House & Senate Independent Bipartisan Citizens Commissions and Missouri Judicial Redistricting Commission

2021–2022 Member of OPEN-Maps Faculty Working Group

2019 *Amicus Curiae*, *Rucho v. Common Cause*, Supreme Court of the United States